Forthcoming in the ***Journal of Financial Economics***

Productivity-Based Asset Pricing: Theory and Evidence *

First Presented: October 2004 FMA Meetings, New Orleans.
Current Draft: August 23, 2006

Ronald J. Balvers **
Division of Economics and Finance
P.O. Box 6025
West Virginia University
Morgantown, WV 26506-6025
Phone: (304) 293-7880
Fax: (304) 293-2233
Email: rbalvers@wvu.edu

Dayong Huang
Division of Economics and Finance
P.O. Box 6025
West Virginia University
Morgantown, WV 26506-6025
Phone: (304) 293-7875
Fax: (304) 293-2233
Email: dahuang@mail.wvu.edu

ABSTRACT

In a general real business cycle model a pricing kernel is derived that involves only production function arguments. The productivity shock is the single factor with as conditioning variable the capital stock relative to a productivity measure. The model compares favorably with the complementary consumption-based and market-based approaches and with the Fama-French three-factor model. A size premium arises from differences in unconditional sensitivities—small firms are more sensitive to productivity shocks—and a value premium from differences in conditional sensitivities to productivity shocks—growth firms are more sensitive to productivity shocks in states when the productivity risk premium is low.

JEL classification: G12; E44

***Keywords*:** Cross Sectional Asset Pricing; Productivity; Macro Factors; Production-Based Asset Pricing; Conditional Asset Pricing.

* The authors thank seminar participants at West Virginia University, the New York Institute of Technology, and the 2004 FMA Meetings, and Ashok Abbott, Arabinda Basistha, Strat Douglas, Alexei Egorov, Sadayuki Ono, Yangru Wu, and especially the anonymous referee, for important comments. Michael Glenn and Marcy Jacobs at the Bureau of Economic Analysis provided valuable data assistance. The authors remain solely responsible for possible errors or omissions.

** Corresponding author.

Productivity-Based Asset Pricing: Theory and Evidence

ABSTRACT

In a general real business cycle model a pricing kernel is derived that involves only production function arguments. The productivity shock is the single factor with as conditioning variable the capital stock relative to a productivity measure. The model compares favorably with the complementary consumption-based and market-based approaches and with the Fama-French three-factor model. A size premium arises from differences in unconditional sensitivities—small firms are more sensitive to productivity shocks—and a value premium from differences in conditional sensitivities to productivity shocks—growth firms are more sensitive to productivity shocks in states when the productivity risk premium is low.

JEL classification: G12; E44

Keywords: Cross Sectional Asset Pricing; Productivity; Macro Factors; Production-Based Asset Pricing; Conditional Asset Pricing.

Date: August 23, 2006

1. Introduction

The Consumption CAPM of Breeden (1979) is the premier theory explaining return variations across assets from an optimizing intertemporal perspective, involving a pricing kernel related to the marginal utility of consumption. The consumption-based approach however faces three major empirical challenges in explaining asset prices: (1) The equity premium puzzle – variation in the marginal utility of consumption appears insufficient to explain the average level of excess returns (Mehra and Prescott, 1985); (2) The almost complete absence of cross sectional explanatory power – the consumption growth rate's lack of explanatory power for excess returns across asset classes (Mankiw and Shapiro, 1986); and (3) The low covariance between risk premium and betas – the covariance between consumption (market) betas and the consumption (market) risk premium obtained from a series of estimates over small time windows is too small to support the importance of any conditioning variable (Lewellen and Nagel, 2005).

A broad explanation for the poor empirical performance of the CCAPM is that the marginal utility of consumption depends not only on consumption but also on an additional variable "*x*". This variable may be a state variable yielding a model with consumption growth as the factor, in which risk premium and betas are conditional on the state variable. Prominent conditional models are Campbell and Cochrane (1999) using lagged consumption as the state variable and Lettau and Ludvigson (2001a, 2001b) employing the consumption-to-wealth ratio. These conditional models answer for much of the CCAPM's poor performance but are subject to the Lewellen and Nagel critique.

The "*x*" variable may also be a contemporaneous variable, in which case we typically obtain an unconditional multi-factor model for which the Lewellen and Nagel critique does not apply. Examples are Parker and Julliard (2005) considering x to be a measurement error; Abel (1990) adding a conspicuous consumption effect; Balvers and Huang (2004) considering a model where money appears in the utility function; Piasezzi, Schneider, and Tuzel (2003) and Yogo (2003) allowing for durable consumption goods; and the formal model below, incorporating a preference for leisure. Evidently, even if we assume the functional form of the utility function, there is no consensus on how to measure the marginal utility of consumption: any combination of the proposed x variables may have a quantitatively significant impact on marginal utility.

A second approach, complementary to the CCAPM was developed earlier by Merton (1973) and has as its pricing kernel the marginal utility of wealth. Its advantage is that changes in wealth can be related directly to market returns. The marginal utility of wealth is also affected by state variables that summarize how valuable wealth is in different states, but the Merton model has little to say about which state variables should be important: in general, any variables affecting future risk or risk aversion are candidates.[1] These would include at a minimum aggregate demand factors such as the variables governing habit persistence and the aggregate supply variables affecting productivity. The Merton approach, insofar as producing a conditional version of the CAPM, is also subject to the Lewellen and Nagel (2005) critique.

A third complementary asset pricing approach looks on the production side instead of the consumption side. It includes quite different contributions. Based on the asset pricing models of Brock (1982) and Lucas (1978), respectively, Balvers, Cosimano, and McDonald (1990) and

[1] Petkova and Zhang (2005) in a recent application of the Merton approach reasonably pick the expected market risk premium as the state variable. By conditioning the expected market risk premium on the term spread, default spread, and lagged risk free rate they are able to explain a substantial component of the value premium. Santos and Veronesi (2006) use an income distribution measure as a state variable in a conditional CAPM.

Cechetti, Lam, and Mark (1990) argue that aggregate output is equal or proportionate to aggregate consumption and that one could evaluate the marginal utility of consumption at the observed level of output so that aggregate output growth becomes the key asset pricing factor.[2] The advantage is that output growth is likely measured more accurately than consumption growth.

Within this third approach, a different production-based perspective is provided by Cochrane (1991, 1996) who explicitly derives an expression for investment returns and argues that these can serve as a pricing kernel for asset returns. Cochrane (1996) finds that investment returns (using two factors as proxies: residential and non-residential investment growth) are significantly priced. Li, Vassalou and Xing (2004) use a more disaggregated model and find that a four-factor investment-growth approach increases the model fit dramatically.[3]

In a recent survey article, Cochrane (2005) argues for the production-based approach relative to the consumption-based approach, stating that "This approach should allow us to link stock returns to genuine business cycle variables, and firms may do a better job of optimization (more precisely, information and transactions cost frictions which we abstract from may be less

---

[2] See Liew and Vassalou (2000) and Vassalou (2003) for recent applications of this approach to cross-sectional asset pricing.

[3] Peng and Shawky (1999) employ Cochrane's (1991) approach to study the equity premium but impose an ad hoc pricing kernel. A recent paper by Jermann (2005) develops a generalized version of Cochrane's (1991) model to consider the size of the equity premium.

Additionally, there are asset pricing models that include a production sector but focus predominately on explaining the size of the equity premium. Jermann (1998) argues that we need both adjustment cost and habit persistence to match the observed equity premium. Boldrin, Christiano, and Fisher (2001) emphasize limited intersectoral factor mobility and habit persistence in order to explain several aspects of the equity premium and key facts of the overall economy. The path breaking model by Gomes, Kogan, and Zhang (2003) provides a structural theory of the risk sensitivities of firms differing by value and size attributes. These approaches focusing on the equity premium belong to the consumption-based approach, rather than the production-based approach, as they use the marginal rate of intertemporal substitution (with or without a habit factor) as the pricing kernel.

Kim (2003) provides a different theoretical perspective by using duality theory in Cochrane's (1996) framework. Vassalou and Apedjinou (2003) empirically develop a corporate innovation factor – a scaled, firm-specific version of Solow's residual – showing that this factor, when added to the market factor, absorbs the momentum effect in cross-sectional asset prices.

important for firms)." (Cochrane, 2005, p. 33). He then adds that to date no pure production-based approach has been developed "…which links macro variables to asset returns *independently* of preferences" (Cochrane, 2005, p.34, italics in original). Our model yields a pure production-based approach in the sense that the variables determining the pricing kernel characterize the production capacity of the economy and are not related to preferences.

Theoretically, we can find that the pricing kernel prices investment returns either based on arbitrage, as in Cochrane (1991, 1996), or based on firms' first-order conditions. Thus, $E_t(m_{t+1} r_{t+1}^I) = 1$, where m_t is the pricing kernel at time *t* and r_t^I is the time *t* investment return for the representative firm. It follows tautologically that $m_{t+1} = \eta_{t+1} / r_{t+1}^I$, with $E_t(\eta_{t+1}) = 1$. This motivates taking investment returns to represent the pricing kernel as in Cochrane (1996) and Li, Vassalou and Xing (2004). However, in principle, other than in mean, η_{t+1} is unrestricted and may represent any factor affecting the pricing kernel in addition to investment returns. We relate η_{t+1} to shocks in the marginal value of capital, obtaining an explicit production-based expression for the pricing kernel.

While our paper fits within the third asset pricing approach, it is thus fundamentally different from the other production-based asset pricing papers. Essentially, we take advantage of the fact that, in a competitive economy with complete financial markets, the marginal rate of intertemporal substitution is tied to a stochastic version of the marginal rate of intertemporal transformation. The former is the pricing kernel in the consumption-based model; the latter becomes the pricing kernel in our model. The factors in the production-based kernel are easily observable and are given uncontroversially, at least in the neoclassical tradition, by labor, capital and productivity shocks. We replace the consumption-based kernel $u_c(c_{t+1}, x_{t+1})$ by a production-based kernel η_{t+1} / r^I_{t+1} and find that the latter performs very well. Our argument in support of the CCAPM thus is that it works well, once we circumvent the problems inherent in measuring marginal utility, $u_c(c, x)$, that are bound to occur when we have only a vague idea of what "*x*" is.

The assumptions in our model of a perfect competition, complete markets, neoclassical environment where supply shocks in the form of productivity shocks are important, brings us close to the canonical version of the Real Business Cycle (RBC) literature. In addition to supporting the CCAPM, our results may be interpreted as using alternative evidence, stemming from the cross section of financial returns, on whether productivity shocks derived from the Solow residual are indeed a key source of macroeconomic fluctuations. If so, the productivity shocks should be able to explain a significant fraction of cross sectional variation in asset returns.

In our objective of assessing how well the traditional RBC model explains the observed cross section of asset prices we overlap with Zhang (2005). He asks whether a specific neoclassical model with asymmetric capital adjustment costs can explain the returns of value and growth firms. The approaches are complementary. On the one hand, our approach derives the pricing kernel in general equilibrium, endogenously generating the appropriate *risk premiums* in the economy; on the other hand, Zhang's approach calibrates an industry equilibrium framework with an exogenous pricing kernel, providing a detailed look at firm-specific aspects, endogenously generating *risk sensitivities*. Further differences are that we focus on explaining the mean excess returns of any financial asset, not only the value premium, and in our specific explanation of the value premium control for size. Our approach also allows conventional evaluation of the model's performance against that of alternative asset pricing models. In addition, the results are not contingent on a specific parameterization or the absence of capital adjustment costs but apply to a large class of RBC models.

The model generates a simple "productivity-based" asset pricing equation with the aggregate productivity shock as the factor and the current aggregate capital stock and level of productivity as conditioning variables, capturing in essence the marginal value of capital. The two state variables appear to be cointegrated, suggesting a parsimonious version of the model with the cointegrating residual becoming the single state variable. The parsimonious model like the original produces reasonable estimates for the level of the productivity risk premium and its conditional variation and generates a covariance between productivity risk premium and betas of sufficient magnitude to address the Lewellen-Nagel critique.

The model results are further consistent with a simple Cobb-Douglas aggregate production function with a productivity parameter in the typical range, although a full-blown calibration of our general equilibrium model is outside the scope of the paper. The model also performs extremely well against the leading alternatives, suggesting not only that the basic RBC framework is consistent with the cross section of asset returns but also that a production-based pricing kernel may be preferable in practice to the consumption-based kernel—presumably because capital and its productivity can be measured more accurately than the marginal utility of consumption. Furthermore, we check the prediction that a proper state variable should forecast returns. We find indeed that, in the typical forecasting regressions, the state variable predicts market returns and the value premium well. Additionally, for the state variable to be priced into expected returns as we find, a further confirmed prediction is that the state variable forecasts returns nonlinearly.

The strong performance of the model in explaining the return difference among the 25 Fama and French (1996) portfolios sorted by size and value arises because the *unconditional* part of the model explains much of the dispersion of mean returns by size class while the *conditional* part explains much of the dispersion of mean returns between value and growth firms: smaller firms have higher exposure to productivity risk on average while, for given size, value firms are most risky in states where the productivity risk premium is higher.

In the remainder of the paper we derive in section 2 the conditional asset pricing model that arises from a standard RBC model and discuss the theoretical implications. Section 3 considers data and methodology. The implications of the theory are examined in section 4 which presents the qualitative and quantitative results and a comparison with competing models. Section 5 concludes.

2. The Model

(a) Derivation of the production-based pricing kernel

First consider that in standard general equilibrium models, including the model below, for every tradable asset return, r_{t+1}^{i}:

$$u_c(c_t, \overline{n} - n_t) = \beta E_t[r_{t+1}^{i}\ u_c(c_{t+1}, \overline{n} - n_{t+1})], \qquad (1)$$

where all subscripts (except subscript *t*, indicating time) represent partial derivatives and β represents the discount factor. For a representative consumer, utility depends on consumption c_t and leisure $\overline{n} - n_t$ (the latter variable being typically ignored in consumption based asset pricing models under the assumption that preferences are separable in consumption and leisure). Equation (1) states that each asset is priced by the pricing kernel (or stochastic discount factor), m_{t+1}:

$$m_{t+1} = \beta u_c(c_{t+1}, \bar{n} - n_{t+1}) / u_c(c_t, \bar{n} - n_t). \tag{2}$$

We employ the standard perfect competition, complete markets general equilibrium model that is used as the work horse in traditional RBC models. See for instance King and Rebelo (2000) for a survey. In this model the equilibrium is Pareto Efficient and a social planner approach can be used to analyze the model properties. The social planner maximizes:

$$V(k_t, \theta_t) = \underset{n_t, k_{t+1}}{Max} \{u(c_t, \bar{n} - n_t) + \beta E_t[V(k_{t+1}, \theta_{t+1})]\} \tag{3}$$

Subject to: $\theta_{t+1} = H(\theta_t, \varepsilon_{t+1}),$ (4)

$$c_t = F(\theta_t, n_t, k_t) - k_{t+1}. \tag{5}$$

The life time utility of the representative consumer is maximized subject to a standard production function $F(\theta_t, n_t, k_t)$ where the inputs are (per-capita) labor n_t and capital k_t, and where production depends also on an exogenous technology/productivity level θ_t. The productivity level is assumed to follow a Markov process as given by equation (4), with ε_{t+1} representing the zero-mean, white noise productivity shock. Per-capita consumption is given in equation (5) as the part of per-capita production that is not invested. The "time-to-build" feature of the model implies that the capital stock is chosen a period before it becomes productive. Labor can be adjusted instantaneously.

These assumptions imply that there are exactly two state variables in the model: the current per-capita capital level k_t and the current productivity level θ_t. Thus, the Bellman equation of dynamic programming is as given in equation (3), where $V(k_t, \theta_t)$ represents the indirect utility function—the maximal lifetime value of utility for the representative consumer—as dependent on the current state of the economy.

It is possible to generalize the above model substantially without changing the state variables. We can add both a time trend and capital dependence to the evolution of the productivity level in equation (4): $\theta_{t+1} = H(\theta_t, k_t, t, \varepsilon_{t+1})$. This would capture, respectively, a deterministic trend in productivity and a "learning by doing" effect whereby a larger capital stock facilitates learning about the productive use of it (see for instance Bakh and Gort, 1993). Investment in equation (5) can be replaced by a more general transformation function, taking capital adjustment costs into account: $c_t = f(\theta_t, n_t, k_t) - g(k_{t+1}, k_t)$, where g is increasing convex in k_{t+1} and decreasing in k_t and is typically assumed to be homogeneous of degree one in its arguments. Note that $F(\theta_t, n_t, k_t) \equiv f(\theta_t, n_t, k_t) + (1-\delta)k_t$ and that without capital adjustment costs $g(k_{t+1}, k_t) = k_{t+1} - (1-\delta)k_t$, where δ represents the depreciation rate of the capital stock.

With the generalizations, the state variables of the model remain k_t and θ_t. Even the deterministic trend t does not become an additional state variable if the model's variables are properly detrended as shown for instance by King and Rebelo (2000, pp. 944-946). Because these generalizations do not change the state variables, and the productivity shock ε_{t+1} remains the only random variable, the cross sectional asset pricing implications are similar. We choose to present the simpler standard RBC model allowing us to argue that the standard model alone is sufficient for generating strong cross sectional explanatory power (resolving the Mankiw-Shapiro issue) and a strong enough covariance between betas and risk premium (resolving the Lewellen-Nagel issue). Further, the interpretation of the empirical results does not rely on the existence of capital adjustment costs. Our approach does not address the equity premium puzzle (the Mehra-Prescott issue). The simple RBC model may well also explain the size of the equity premium but it is not unlikely that the more general version of the model, with capital adjustment costs as in Cochrane (1991, 1996) and Zhang (2005), would be essential for this purpose.

The first-order conditions for the capital and labor choices are:

$$u_c(c_t, \bar{n} - n_t) = \beta E_t[V_k(k_{t+1}, \theta_{t+1})], \tag{6}$$

$$u_n(c_t, \bar{n} - n_t) = u_c(c_t, \bar{n} - n_t)\, F_n(\theta_t, k_t, n_t). \tag{7}$$

The envelope condition is

$$V_k(k_t, \theta_t) = u_c(c_t, \bar{n} - n_t)\, F_k(\theta_t, n_t, k_t) \tag{8}$$

Update equation (8) by one period and combine with equation (6) to yield:

$$\frac{\beta\, u_c(c_{t+1}, \bar{n} - n_{t+1})}{u_c(c_t, \bar{n} - n_t)} = \left(\frac{V_k(k_{t+1}, \theta_{t+1})}{E_t V_k(k_{t+1}, \theta_{t+1})}\right)\left(\frac{1}{F_k(\theta_{t+1}, n_{t+1}, k_{t+1}}\right) \tag{9}$$

Thus, from equations (2) and (9) we obtain a production-based pricing kernel:

$$m_{t+1} = \eta_{t+1} / F_k(\theta_{t+1}, n_{t+1}, k_{t+1}), \quad \eta_{t+1} = V_k(k_{t+1}, \theta_{t+1}) / E_t[V_k(k_{t+1}, \theta_{t+1})]. \tag{10}$$

Note that in our context without adjustment costs we have that the marginal product of capital $F_k(\theta_{t+1}, n_{t+1}, k_{t+1})$ is exactly the investment return r_{t+1}^I and the marginal rate of intertemporal transformation (MRIT) equals $1/F_k(\theta_{t+1}, n_{t+1}, k_{t+1})$ and differs from the pricing kernel by a factor

η_{t+1} with $E_t \eta_{t+1} = 1$. The reason that MRIS ≠ MRIT state by state though markets are complete is that there is an intertemporal risk that even under perfect risk sharing cannot be avoided – given time to build, the uncertainty about the productivity shock has real implications that are unavoidable in the aggregate. The η_{t+1} term represents a fundamental, nondiversifiable risk inherent in capital accumulation that cannot be ignored and should be essential for asset pricing. The productivity-based pricing kernel thus consists of two components: the MRIT term accounting for how much additional capital can be produced physically out of (real) financial payoffs, and the η_{t+1} term accounting for the unanticipated change in the marginal valuation of this additional capital.

(b) Variables in the Productivity-Based Pricing Kernel

We examine whether the pricing kernel, derived from a model that has been successful in characterizing key macroeconomic moments, is also useful in pricing the cross section of financial assets. We focus on the case of excess returns for two reasons. First, since the kernel is in real terms it prices in real terms and returns need to be corrected for inflation. Rather than using some inflation measure we subtract a benchmark asset return that is also in nominal terms so that the resulting return is automatically adjusted for inflation. Second, the pricing kernel for excess returns (and more generally zero-investment portfolios) can be simplified, leading to a more parsimonious factor specification.[4] The pricing equation for excess returns is:

---

[4] Based on Cochrane's (1996) linear conditioning approach the *excess* returns case leads to a two-factor or a three-factor conditional model, depending on whether one or two state variables are employed. When instead *real* returns are considered, we end up with three or five factors, respectively. The difference occurs because the state variables by themselves become factors affecting the unconditional mean level of returns but not excess returns.

$$E_t[r_{t+1}^{iE}\, V_k(k_{t+1},\theta_{t+1}) / F_k(\theta_{t+1}, n_{t+1}, k_{t+1})] = 0, \text{ for all } i, \tag{11}$$

where in this excess returns formulation we can eliminate the $E_t[V_k(k_{t+1},\theta_{t+1})]$ term in the pricing kernel. It is easy to see that k_{t+1} depends on the current capital level k_t and the current productivity level θ_t only, whereas (from equation 7) n_{t+1} depends on k_{t+1} and θ_{t+1}. The latter, in turn, depends on θ_t and the shock ε_{t+1} as follows from equation (4). Thus:

$$m_{t+1}^E = V_k(k_{t+1},\theta_{t+1}) / F_k(\theta_{t+1}, n_{t+1}, k_{t+1}) = M(k_t, \theta_t, \varepsilon_{t+1}). \tag{12}$$

Equation (12) implies that all returns can be explained by k_t, θ_t, and ε_{t+1}. Conditional on information at time *t*, solely the productivity shock ε_{t+1} matters, so this is the single factor in pricing all assets. However, the extent to which ε_{t+1} matters depends on the state of the economy and is determined by k_t and θ_t.

In principle, the production-based pricing kernel in equation (12) should price assets exactly as well as the consumption-based pricing kernel in equation (2). However, as pointed out by Campbell (1993), the necessary consumption data may not be measured well and, as argued above, the consumption-based kernel may include additional mysterious "*x*" variables that are not easily identified. Thus, application of the consumption-based pricing kernel is not straightforward. We propose to evaluate the usefulness of the production-based kernel empirically. While others have looked at production-based asset pricing, their work is methodologically very different. Balvers, Cosimano, and McDonald (1990) and Cechetti, Lam, and Mark (1990) merely replace consumption by output and do not even consider cross sectional asset pricing implications. Cochrane (1996) and Zhang (2005) impose a particular production-based pricing kernel exogenously.[5]

(c) The Conditional Beta Expression

Assume that all returns and the productivity shock are jointly conditionally normally distributed. This assumption is not essential but is sufficient to allow us to linearize the pricing kernel. From equations (11) and (12), using the definition of covariance:

$$E_t(r_{t+1}^{iE})\, E_t[M(k_t,\theta_t,\varepsilon_{t+1})] + Cov_t[r_{t+1}^{i}, M(k_t,\theta_t,\varepsilon_{t+1})] = 0\,. \tag{13}$$

Using Stein's Lemma (which applies given the conditional multivariate normality assumption):

$$E_t(r_{t+1}^{iE}) = \frac{-E_t[M_\varepsilon(k_t,\theta_t,\varepsilon_{t+1})]}{E_t[M(k_t,\theta_t,\varepsilon_{t+1})]} Cov_t(r_{t+1}^{i},\varepsilon_{t+1})\,, \tag{14}$$

Defining:

$$b(k_t,\theta_t) = \frac{E_t[M_\varepsilon(k_t,\theta_t,\varepsilon_{t+1})]}{E_t[M(k_t,\theta_t,\varepsilon_{t+1})]}\,, \tag{15}$$

and given $E_t(\varepsilon_{t+1}) = 0$ we have from equation (14) that

$$E_t\{\, r_{t+1}^{iE}\, [1 + b(k_t,\theta_t)\,\varepsilon_{t+1}]\} = 0\,, \text{ for all } i. \tag{16}$$

[5] Cochrane (1996) explicitly derives an expression for investment returns and then assumes that a subset of the investment returns is a good approximation for the pricing kernel.

Equation (16) implies a "mimicking" linear pricing kernel that prices all zero-investment asset portfolios equally as well as the original kernel in equation (12):

$$M^{*}(k_t,\theta_t,\varepsilon_{t+1}) = 1 + b(k_t,\theta_t)\,\varepsilon_{t+1}, \tag{17}$$

Equation (16) gives directly an expression for conditional mean excess returns:

$$E_t(r_{t+1}^{iE}) = -b(k_t,\theta_t)\,Cov_t(r_{t+1}^{i},\varepsilon_{t+1}). \tag{18}$$

If we construct a zero-investment position with *unit sensitivity* to the productivity shock, its conditional expected return $E_t r_{t+1}^{\varepsilon}$ can be interpreted as the productivity risk premium. Applying equation (18) it is given as

$$RP_t \equiv E_t(r_{t+1}^{\varepsilon}) = -b(k_t,\theta_t)\,Var(\varepsilon_{t+1}), \tag{19}$$

where the white noise properties of ε_{t+1} imply that the unconditional variance of ε_{t+1} equals the conditional variance. Eliminating $b(k_t,\theta_t)$ from equations (18) and (19) gives the beta formulation $E_t(r_{t+1}^{iE}) = \beta_t^{i\varepsilon}\,E_t(r_{t+1}^{\varepsilon})$.

We focus empirically on the unconditional implications of this conditional model. Take unconditional expectations in equation (16) to obtain

$$E\{r_{t+1}^{iE}[1 + b(k_t,\theta_t)\,\varepsilon_{t+1}]\} = 0, \text{ for all } i. \tag{20}$$

Then unconditional means are given as

$$E(r_{t+1}^{iE}) = -E(b_t)Cov(\varepsilon_{t+1}, r_{t+1}^{iE}) + Cov(RP_t, \beta_t^{i\varepsilon}), \tag{21}$$

where $b_t = b(k_t, \theta_t)$, $\beta_t^{i\varepsilon} = Cov_t(r_{t+1}^i, \varepsilon_{t+1}) / Var(\varepsilon_t)$, and $RP_t = -b_t \sigma_\varepsilon^2$ from equation (19). The unconditional mean excess return consists here of an unconditional part and a conditional part. The *unconditional* part is the premium based on the productivity risk and asset risk sensitivity in the average state. The *conditional* part arises because of correlation between the productivity risk premium and an asset's productivity beta, its sensitivity to the productivity shock. If an asset's productivity risk exposure is high (high $\beta_t^{i\varepsilon}$) when the risk premium is low (b_t closer to zero) then there is a negative covariance between risk premium and beta which subtracts from the asset's overall riskiness and produces a negative conditional return component.

(d) Return and Factor Stationarity

Accepting theoretical and empirical arguments that asset returns are (second-order) stationary, the pricing kernel must be stationary as well. Equation (20) implies that $b(k_t, \theta_t)$ must be stationary in order for the second moments of returns to be stationary. This may occur in two different cases. Either both k_t, θ_t are trend stationary (i.e., given the potential model interpretation in terms of detrended variables, these variables are stationary once a time trend has been removed) or k_t, θ_t are both nonstationary but are cointegrated so that we may write:

$$b(k_t, \theta_t) = \overline{b}(s_t, \theta_t) = \widehat{b}(s_t) = b^0 + b^1 s_t, \tag{22}$$

where s_t is the, by definition, stationary cointegrating residual of a regression of k_t on θ_t and a constant. The first equality follows tautologically whereas the second equality implies a restriction that must hold if the pricing kernel is to be stationary. Equation (22) then allows us to generate a singular state variable that is sufficient to capture the state of the economy and is found as the cointegrating residual based on the two original state variables. The third equality holds because, for empirical purposes, we take a first-order approximation around the mean of the state s_t, with b^0, b^1 constant.

(e) Implications

The signs of the parameters in the linearized pricing kernel, substituting equation (22) into equation (17), can be determined by straightforward comparative statics analysis based on equation (12). The signs are formally ambiguous because of the typical opposing income and substitution effects. For instance, the effect of a change in ε_{t+1} in equation (12) depends on how ε_{t+1} affects the marginal value of capital for the indirect lifetime utility of the representative consumer compared to how capital affects current production at the margin. Both effects are positive and are difficult to compare quantitatively. From equations (8) and (12) we may instead look at how ε_{t+1} affects c_{t+1}. Here the opposing effects are more intuitive: One effect is a direct increase in production for given inputs, leading via the income effect to higher consumption and a lower marginal utility of consumption. The second effect is the substitution effect of investment becoming more productive, causing consumption to fall for given production.

If the investment elasticity is not too large then the income effect dominates so that higher

ε_{t+1} is associated with higher c_{t+1} and lower marginal utility of consumption.[6] This implies that b^0 is *negative*: excess returns are discounted more strongly (valued less) in the face of productivity risk because a positive shock leads to more consumption and a lower marginal evaluation of returns generated in this case. The factor risk premium for productivity risk then is positive from equation (19) and assets that load heavily on the productivity shock factor require high average returns.

The investment elasticity, however, fluctuates with the state of the economy, which explains the role of the state in affecting asset prices. In general the effect of the state variable is complex and difficult to assess a priori. Anticipating that the cointegrating regression yields a positive link between k_t and θ_t (because a higher productivity level leads eventually to a higher capital stock), a positive cointegrating residual, capturing the state s_t of the economy, indicates a surplus of k_t relative to θ_t. This surplus may be viewed as an excess of tangible capital relative to intangible capital. Investment (in tangible capital) will generally be low in this state but a given positive productivity shock ε_{t+1}, enhancing intangible capital, may well have a larger impact on the marginal value of capital when intangible capital is relatively scarcer, i.e., s_t is higher, inducing more investment. Investment becomes more sensitive and, accordingly, consumption less sensitive to ε_{t+1}. So productivity shocks have less impact on the marginal utility of consumption, making them less risky. In this scenario we expect b^1 to be *positive*, so that an increase in the state variable implies that $b(k_t, \theta_t)$ is closer to zero. Thus the risk premium on the productivity factor

[6] To simplify the intuitive discussion, we ignore here the effect that leisure has on the marginal utility of consumption. A positive productivity shock leads to increased labor demand and less leisure. This may increase or decrease the marginal utility on consumption depending on whether leisure and consumption are substitutes or complements.

varies over time, with increases in the state of the economy mitigating the role of productivity risk on consumption and lowering the productivity risk premium.

Given equation (22), equation (21) can be written as a more explicit two-factor model, with factors $s_t\varepsilon_{t+1}$ and ε_{t+1} and intercept *a*:

$$E(r_{t+1}^{iE}) = a - b^0 \, Cov(\varepsilon_{t+1}, r_{t+1}^{iE}) - b^1 \, Cov(s_t\varepsilon_{t+1}, r_{t+1}^{iE}) \,. \tag{23}$$

From the theory we expect to find $a = 0,\ b^0 < 0,\ b^1 > 0$. In addition to the slope coefficients, the intercept is important since a positive (say) intercept implies that the risk factors systematically underpredict the mean returns of individual assets, or, in other words, that quantitatively the risk premia are too small to explain the mean level of excess returns. A non-zero intercept can also be interpreted as the model's pricing error in pricing any zero-beta asset (its excess return theoretically should be zero but is predicted to be *a*).

An additional implication is that, in principle, based on equation (23) the factors $s_t\varepsilon_{t+1}$ and ε_{t+1} explain *all* of the cross sectional variation in unconditional mean asset returns. Thus, the cross sectional fit is an important criterion for determining the adequacy of the model. In evaluating the cross sectional fit we use the *adjusted R-square* calculated from the actual returns and predicted returns. It is more appropriate than Hansen's (1982) *JT statistic*, if the question is not so much whether the asset pricing model used is actually true but instead whether the model explains a large fraction of cross sectional differences in asset returns.

Equation (21) provides a quantitative check on the importance of the state variable.

Lewellen and Nagel (2005) raised the issue that estimating the parameters of a linear pricing kernel to best fit a particular set of mean asset returns does not exhaust the empirical implications because, while the coefficients are set to match cross sectional variation, there is no guarantee that the coefficients match quantitatively the magnitude of the covariance between beta and risk premium across states. From equation (21) $dE(r_{t+1}^{iE})/dCov(RP_t\,,\,\beta_t^{i\varepsilon})=1$

A further unconditional implication of the model follows from the fact that the pricing kernel for assets must also price aggregate investment returns: the pricing kernel in equation (10) $m_{t+1}=\eta_{t+1}/F_k(\theta_{t+1},n_{t+1},k_{t+1})$ implies that $E_t[m_{t+1}F_k(\theta_{t+1},n_{t+1},k_{t+1})]=1$ because $E_t\eta_{t+1}=1$. This relation allows us to establish whether the pricing kernel that we identify empirically implies quantitatively appropriate investment returns. Specifically, if we make the typical assumption of a Cobb-Douglas production function, then we have $F_k(\theta_{t+1},n_{t+1},k_{t+1})=\alpha\,(y_{t+1}/k_{t+1})+1-\delta$. To consider excess returns subtract $E_t[m_{t+1}(1+r_{t+1}^f)]=1$, with r_{t+1}^f the real riskfree rate. It follows that $\alpha=E[m_{t+1}(\delta+r_{t+1}^f)]/E[m_{t+1}(y_{t+1}/k_{t+1})]$, which we can compute given the empirical time series for the pricing kernel.

We also consider empirically two conditional implications of the model. In addition to the productivity risk premium, the sensitivities of individual asset returns to the productivity shock generally should be time-varying as well because, depending on the particular asset, the state of the economy also affects how the productivity shock impacts particular firms and industries. As in Lettau and Ludvigson (2001b): [7]

[7] We have that $r_{t+1}^{iE}=E(r_{t+1}^{iE})+\beta^{i0}\varepsilon_{t+1}+\beta^{i1}\,s_t\varepsilon_{t+1}+\eta_{t+1}$. If ε_{t+1} and η_{t+1} are independent, equation (24) follows.

$$\beta_t^{i\varepsilon} = \beta^{i0} + \beta^{i1} s_t \,. \tag{24}$$

This implies, if the state indeed has an important impact on unconditional mean returns (b^1 not equal to zero), that not only should the state forecast future returns (as emphasized by Ferson and Harvey, 1989, Campbell, 1996, Lettau and Ludvigson, 2001a, and Santos and Versonesi, 2006, following Merton 1973), but it should do so quadratically. From equations (16), (22), and (24) we obtain:

$$E_t(\, r_{t+1}^{iE}) = -\sigma_\varepsilon^2 \,[b^0\, \beta^{i0} + (b^0\, \beta^{i1} + b^1\, \beta^{i0})\, s_t + b^1 \beta^{i1} s_t^2\,]\,. \tag{25}$$

Since the state forecasts next-period's mean returns, it should be correlated with future realized returns. Taking unconditional expectations in equation (25) it follows because $E(s_t) = 0$ that mean returns depend on their state sensitivities through the quadratic term only. The state variable s_t forecasts returns and does so quadratically if the conditional variable is to explain mean return differences. Although we make specific linearity assumptions this implication illustrates the more general principle that any effect of state variables on unconditional mean returns must result from nonlinearities. Given b^1 positive, firms i with positive β^{i1} have lower mean returns since their positive loading on $s_t \varepsilon_{t+1}$ implies that they have their highest productivity risk when the productivity risk premium is lowest.

3. Empirical Preliminaries

The model implies that asset returns are determined by one systematic factor: the aggregate productivity shock, with the cointegrating residual measuring the size of the capital stock relative to the productivity level as the conditioning variable. We discuss next the methodology and the data used to evaluate the model.

(a) Methodology

The method employed is mostly the one-stage Generalized Method of Moments (GMM) estimation of Hansen (1982). We focus on one-stage GMM estimation because as recommended by Altonji and Segal (1996), Lettau and Ludvigson (2001b) and Cochrane (2001), Hansen's optimal weighting matrix is poorly estimated when the cross section of test assets is large relative to the time series of data, which is the case in our sample. Jagannathan and Wang (2002) have shown that the stochastic discount factor (pricing kernel) approach is as efficient as the expected return-beta approach and that it has higher power in detecting model misspecifications. Furthermore, one-stage GMM estimation enables us to focus on economically interesting portfolios. We follow Cochrane (2001) and formulate asset returns in terms of the pricing kernel and apply one-stage GMM to find the pricing kernel's loadings on the risk factors. With these loadings, we then compute the implied risk premia.[8] To evaluate the overall statistical significance of the model, we rely on Hansen's (1982) JT statistic.

(b) The data

Our main test assets are the 25 portfolios sorted by market capitalization and book-to-market ratio. The returns for these assets are available from Kenneth French's website. To construct excess returns, we subtract the (nominal) three-month T-bill rate. Figure 1(a)

[8] The risk premia using one-stage GMM are very close to those obtained by the Fama and MacBeth (1973) method.

illustrates the unconditional mean returns of the portfolios separated by size (market capitalization) and value (book-to-market ratio).

We construct the productivity level θ_t following King and Rebelo (2000).[9] In the aggregate production function, labor inputs have a weight of 2/3 and capital has a share of 1/3. We obtain Population, Labor Hours, Gross Domestic Investment, and Capital data directly from the Bureau of Economic Analysis. Other data are from the Federal Reserve Bank of St. Louis. Our sample starts 1964Q1, restricted by the limited availability of data on labor hours, and ends 2004Q4. Quarterly capital stock data are intrapolated from annual using the quarterly investment data and the annual capital stock measure, with year-end 1963 as the starting point: for each quarter a fraction of the annual capita increment is added to the current end-of-year stock, with the fraction given as the year's investment to date divided by total investment. We employ per-capita data throughout, dividing by population.

Following King, Plosser, Stock, and Watson (1991) the productivity shock ε_{t+1} for the case where θ_t and k_t are cointegrated is obtained by first differencing log θ_{t+1} and then subtracting the mean. We find that the shock thus constructed is correlated with both lagged state variables θ_t and k_t. Hence, to whiten the shock, we regress it on these variables (or, with virtually identical result as suggested by Sims, Stock, and Watson, 1990, on the cointegrating residual s_t) to obtain the random productivity shock.[10] The regression results are in Table 1 which

[9] Due to data availability, alternative (and less common) measures for productivity shocks (for example, Jorgenson and Stiroh, 1999, and Basu et al., 2000) or productivity shocks adjusted for capital utilization, are not considered.

[10] We make the correction solely to obtain an asset pricing factor that is appropriately random. The dependence on k_t suggests that formally equation (4) should be changed to add k_t on the right-hand side to incorporate a learning by doing effect. However, this would have no impact on the theoretical results. In addition, whether or not we correct the shock for the correlation with the lagged variables has a negligible effect on the empirical results that follow.

also presents further summary statistics for the factor and state variable. Figure 2 displays the time series. Both series fluctuate around a mean of zero and show no time trend. The state variable is highly persistent. The productivity shock shows no persistence but may have some conditional heteroskedasticity which we disregard.

For the no-cointegration case and when the productivity level is presumed trend stationary we adjust k_t for a linear time trend and we find ε_{t+1} as the residual from regressing (in logs) θ_{t+1} on θ_t, a constant, and a linear time trend as in King and Rebelo (2000).[11]

Figure 1(b) depicts the two state variables θ_t and k_t (standardized by subtracting their means and dividing by their standard deviations) over the sample period. Both variables are clearly nonstationary and appear to be cointegrated as is expected theoretically from King et al. (1991) when the productivity level is not stationary. Table 1 displays the results of formal unit root and cointegration tests. The ADF tests for a unit root indicate that a unit root cannot be rejected for either state variable. Proceeding under the assumption that both state variables have a stochastic trend we next check if the trends are cointegrated. The Johansen tests provide clear evidence that the state variables are indeed cointegrated although this is not so clear from the ADF test. In the following we use as our main specification the formulation with the cointegrating residual as the single state variable

4. Results

(a) The Conditional and Unconditional Productivity-Based Models

---

[11] Using instead the HP filter to adjust for the time trend or adding k_t in the regression to whiten the residual has no appreciable impact on the empirical results.

The one-factor (unconditional) model

Table 2 and Figure 3 report the results for the unconditional and conditional versions of our model. The unconditional version follows from equation (20) under the assumption that b_t is constant or from equation (23) assuming that $b^1 = 0$. The GMM estimation based on the 25 Fama-French portfolios sorted by market capitalization and book-to-market ratio yields an estimate of b^0 that is *negative* as expected (given moderate investment elasticity) but not significantly so. It generates a positive quarterly risk premium of *0.36*. Average excess returns in our sample are *9.4%* on an annual basis. With average productivity betas estimated around *4.4*, the productivity risk implies an average annual excess return of approximately *4 x 4.4 x 0.36% = 6.3%.* The unconditional model however has a second-pass alpha of *0.77%* -- *3.1%* annualized -- and a cross sectional fit (R-square) of only *18.2%* (see Figure 3). The JT statistic based on the pricing errors strongly rejects the model with a p-value of *0.000*.

The two-factor (conditional) model

The conditional version of the model in equation (23), however, performs well. The GMM estimation gives $b^0 = \textit{-224.4}$ and $b^1 = \textit{438.36}$ with the expected signs, implying a quarterly productivity risk premium of *0.87%* and a conditional risk premium of *-0.51%.* The Fama-MacBeth t-statistics are significant at *3.12* and *-2.62*, respectively; the Shanken t-statistics, are *1.40* and *-1.17*, respectively.[12] The cross sectional fit is *71.5%* as shown in Figure 3 and the JT

---

[12] As pointed out by Lettau and Ludvigson (2001b) and Santos and Veronesi (2006), the Shanken correction in the t-statistics appears to be more severe for macro factors. Fama-MacBeth t-statistics are also reported because Jagannathan and Wang (1998) show that the Fama-MacBeth procedure may understate rather than overstate the precision of the standard errors under conditional heteroskedasticity.

statistic is *28.85* implying that the model cannot be rejected statistically (*p=0.15*). The model's alpha is *0.47%* which is statistically insignificant and less than *2%* annualized. Given the mean of the productivity beta of *3.9* and a mean conditional factor beta of *3.0*, productivity risk implies an average annual excess return of *13.6%* and the conditional factor implies an average annual excess return of *-6.1%*. In total, the alpha, unconditional risk, and conditional risk premium add up to *1.9 + 13.6 - 6.1 = 9.4%* which is also (as required statistically) the annual equal-weighted mean excess return in our sample.

Conditional and unconditional risk components of the two-factor model

Figure 4 depicts the decomposition of each asset's expected return based on equation (23) by its unconditional $a - b^0\, Cov(\varepsilon_t, r_t^{iE})$ and conditional $-b^1\, Cov(s_{t-1}\varepsilon_t, r_t^{iE})$ components. It shows that the size premium is explained by differences in average exposure to productivity risk: small firms have higher productivity betas $\beta_\varepsilon^i = Cov(\varepsilon_t, r_t^{iE}) / Var(\varepsilon_t)$ than large firms. The value effect is explained by differences in sensitivity $\beta_{s\varepsilon}^i = Cov(s_{t-1}\varepsilon_t, r_t^{iE}) / Var(\varepsilon_t) Var(s_t)$ to the conditional risk factor: growth firms have higher conditional factor betas than value firms so that, given size and the negative conditional factor risk premium $-b^1 Var(\varepsilon_t) Var(s_t)$, mean returns on value firms exceed those of growth firms. Note that conditional factor exposure also explains part of the size premium.

Table 3 lists the individual excess return components for the 10 assets consisting of five size and two value classes. For each size class the *unconditional* return components of growth firms and value firms in Panel B are quite similar; but for each size class the *conditional* return components in Panel C of value firms are for all size classes at least *1%* per quarter more and on average *1.23%* more than for growth firms. This compares reasonably well to the *actual* value premiums for each size class in Panel A which are always positive, varying from *0.49%* to *2.75%*, with an average of *1.54%*.

The model implies from Panels B and C an average size premium (between small and big) of around *1.13%* which is similar to that in the actual data in Panel A of *0.98%*. However, the actual data (see Figure 1 or Table 3, Panel A) indicate a size premium of *2.10%* for value firms and *-0.15%* for growth firms whereas our model implies a size premium that is similar, around *1.20%* for both value and growth firms.

Discussion of the two-factor model results: the conditional risk premium

The negative value for b^0 implies an average positive risk premium of $-b^0\, Var(\varepsilon_t)$. An intuition for the positive productivity risk premium is that positive (negative) productivity shocks coincide with high (low) consumption availability and accordingly low (high) marginal valuations of returns generated in that situation. Assets with positive sensitivity to the productivity shock generate low (high) returns when the marginal valuation of these returns is high (low) and are thus risky, requiring a higher mean return. Alternatively put, an increase in ε_{t+1} affects (lowers) the marginal utility of consumption. This would occur typically when the positive productivity shock raises production but does not increase investment by so much as to offset the increased production, so that in total consumption increases and the marginal utility of consumption falls.

The strength of the productivity shock impact on investment depends however on the state: the higher is s_t the larger is the capital stock in relation to its productivity which affects the incentive to invest. The obtained positive value for b^1 implies that the productivity risk premium falls as the state variable s_t increases. This must occur because a larger part of a production shock is offset by a change in investment causing consumption and marginal utility to be more stable, which is necessary for a smaller risk premium. The positive value of b^1 and its explanation raise a few issues: does, in fact, investment sensitivity to productivity shocks increase with the state and, if so, what may be the reason?

To address whether an increase in the state indeed raises the investment sensitivity to the productivity shocks, we regress (gross) aggregate investment growth against the relevant variables from our model.[13] Panel H in Table 1 shows that the average productivity shock elasticity is around one and that the coefficient on the state variable is significantly negative. The latter negative sign is expected as a higher state corresponds to a lower marginal product of capital. More to the point, investment's productivity shock elasticity increases significantly with the state as evidenced by the positive sign on the cross product of state and productivity shock. Quantitatively, the regression result of Panel H in Table 1 implies that a one standard deviation change in the state, from Panel A in Table 1, changes the elasticity by *1.023 x 0.56 = 0.57%.* Thus, as the state rises from one standard deviation below to one standard deviation above average, the investment elasticity to the productivity shock increases from *0.41* to *1.55.*

To complete the intuitive discussion of the positive sign of b^1 we discuss a possible reason for why the investment elasticity to the productivity shock increases in the state variable. Essentially s_t is a measure of the abundance of tangible capital (the capital stock k_t) relative to

---

[13] Investment includes both nonresidential and residential investment. Results are qualitatively similar, but weaker, when we consider these components separately. The explanatory variables are the state variable, the productivity shock, and their interaction. Note that the state variable does not by itself appear as a determinant of financial asset returns or investment returns since we consider excess returns, causing the predetermined factor to drop out.

intangible capital (the productivity level θ_t). In a "capital abundant" state (when tangible capital is high relative to intangible capital) a positive productivity shock contributes to the relatively scarce intangible capital, closing part of the gap and raising the marginal value of tangible capital to a relatively large degree, which increases investment incentives substantially. In a "capital scarce" state (when tangible capital is low relative to intangible capital) a positive productivity shock contributes less as intangible capital already is abundant and so investment increases little. Note that a capital abundant state by itself implies low investment (since the marginal product of capital is low), as we find empirically in Table 1, Panel H. However, it is the component of investment in response to the unanticipated productivity shocks that affects the consumption risk.

An alternative intuition for why investment elasticity to the productivity shock increases with the state variable is that capital abundance indicates a situation in which the marginal value of (tangible) capital is currently low. Thus, production has less value at the margin which is equivalent to the (shadow) price of current investment being lower. Now a positive productivity shock, raising both future productivity and current output in an environment where investment is "cheap" may well lead to a larger aggregate investment response, even when capital is currently abundant. Similarly, when capital is scarce, expansion is expensive so that a positive shock may have only a small impact on investment.[14] [15]

---

[14] These intuitions may not apply if, instead of absence of adjustment costs, the more realistic assumption of irreversibility of investment is made. With irreversibility the investment elasticity to the productivity shock may become zero at a high enough value for the state variable (capital abundance) when this situation represents an unplanned excess of capital.

[15] A possible alternative explanation for a negative effect of the state variable on the productivity risk premium is that the variance of the productivity shock decreases with the state. To check this we regress the square of the productivity shock on the (lagged) state variable. The effect is close to being statistically significant (Newey-West t-values of 1.76 and 1.79) but the direction of the effect is opposite: if anything, the productivity shock variance increases in the state.

Discussion of the two-factor model results: the value and size premium

From an empirical perspective, why does our conditional productivity-based model explain the value and size-sorted portfolio means better than an unconditional productivity-based model and as well as the Fama-French three-factor model designed specifically to explain these mean returns? The unconditional excess return component is positive for all test assets—all have positive exposure to the productivity factor on average. However, there is a clear pattern in that smaller firms have considerably higher sensitivity to the productivity shock. Thus, the *unconditional* component does well in explaining the size effect.

The *conditional* component is negative for all test assets. It represents the fact that, for all assets, $Cov_t(\varepsilon_{t+1}, r_{t+1}^{iE})$ —the excess return's sensitivity to the productivity shock—is higher when the state of the economy s_t is higher. But a higher state implies a smaller risk premium on the productivity factor, so expected excess returns are lower as equation (23) implies. The key observation in Figure 3 is that the reduction in the mean excess returns based on the conditional component is largest for growth firms. Therefore, the conditional component does well in explaining the value premium: growth firms are on average about as sensitive to productivity shocks as are value firms, but their sensitivity is high for states in which the risk premium is low and consequently they require a lower expected return on average than do value firms.

Zhang (2005) also addresses the value premium focusing on productivity shocks as the risk factor. The value premium in his paper exists because value is riskier than growth when the productivity risk premium is high and in this sense Zhang's account coincides with ours. His interesting explanation is that, due to asymmetric costs of adjustment, in bad times value firms face extra risk since they cannot dispose of their excess capital without considerable cost. This explanation conflicts with our empirical results in two ways when we consider changes in the productivity level (Zhang's state variable and a component of our state variable). In Zhang's model a lower level of productivity *raises* the risk premium and increases productivity betas, for *value* firms more than for growth firms. In our model a lower level of productivity *lowers* the risk premium and increases betas, for *growth* firms more than for value firms (see the productivity betas provided in Table 4). We believe that the discrepancy is related to the omission of capital from Zhang's pricing kernel (and thus from his definition of the state). The inclusion of capital is extremely important. One indication is the empirical importance of capital when it is separated from the productivity level in our three-factor model in Table 2. Additionally, capital moves positively over time with the productivity level (the cointegrating relationship) and the productivity level by itself may be nonstationary, reasons why Zhang's results may be misleading as to the impact of the productivity level in isolation. Further, the difference in the betas of value and growth stocks is likely affected by the aggregate capital stock since "assets in place" for which capital is a proxy have a different degree of importance for value firms compared to growth firms.

Endogenous factor sensitivities: an informal analysis

All assets have a positive unconditional excess return. Given the negative value for b^0 this implies that all assets load positively on productivity risk on average. This is intuitive because a positive aggregate productivity shock should directly benefit most firms. Table 3 also shows that all assets have negative conditional excess return components. This implies from equation (21) or (23) that, for instance, a state with a lower risk premium on ε_{t+1}, is associated with all assets having higher sensitivity to ε_{t+1} (the productivity betas are higher in a capital abundant state), or put alternatively, the firm betas on the conditional factor are all positive. Our exposition takes the betas as exogenous but would be more complete if the systematic movements of betas with the state were also explained. Here our macro-based model is at a disadvantage relative to the recent contributions of Gomes, Kogan, and Zhang (2003) and Zhang (2005) who provide specific models of how factor sensitivities change over time for classes of firms. We can speculate as to plausible causes for the differences in factor exposures that explain the size and value premiums in a way that is consistent with the model but our formal model is not designed to explain these differences.

View firms as convex combinations of two projects, one with immediate payoff and one an option on a technology with future payoff (a growth option). Value firms emphasize the former project; growth firms the latter. A higher current value for the state implies lower investment costs (current and, likely, future), increasing the value of the growth option. At the higher state, a productivity shock just when investment is cheap may have a stronger effect on growth option returns, but this is difficult to assess without a formal model. However, if the productivity shock does have a stronger impact on the growth option return, then all firm productivity betas vary directly with the state but those of growth firms vary more than those of value firms. This is the pattern that we observe. This explanation does not rely on adjustment costs and is accordingly consistent with a narrow interpretation of our model. Table 4 (both panels), discussed more thoroughly in a following section, shows clearly that the productivity betas for all firms increase as the state improves and that, for all size classes, the increases are substantially larger for growth firms than for value firms.[16] A formal examination of these issues awaits future research.

---

[16] An alternative explanation for this observation may be that firms increase financial leverage as (collateralizable) capital becomes more abundant when the state increases, causing equity betas to increase. If this effect is more important for growth firms their betas increase more. Table 4 also shows that small firms have substantially higher

Investment returns in the two-factor model

Given an aggregate Cobb-Douglas production function $f(\theta_t, n_t, k_t) = \theta_t (k_t)^{\alpha} (n_t)^{1-\alpha}$ we can impute the production function parameter from $\alpha = E[m_{t+1}(\delta + r_{t+1}^{f})] / E[m_{t+1}(y_{t+1} / k_{t+1})]$. To compute the Solow residual we use the standard calibration of $\alpha = 0.33$. Matching the aggregate investment return series to the pricing kernel with quarterly depreciation set to δ = 0.025 implies instead $\alpha = 0.24$. Thus, while there is some discrepancy, it appears that our model implies a reasonable value for the Cobb-Douglas production function parameter that is in the ball park of the range of values usually considered for the parameter. (For instance DeJong and Ingram, 2001, obtain $\alpha = 0.23$). Put alternatively, the two-factor model also prices investment returns quite well.

The three-factor (conditional) model

For completeness we present the results of the three-factor model that arises if we do not account for possible stochastic trends and cointegration between the two state variables k_t and θ_t. We now detrend the state variables using linear deterministic trends (results for HP filters are similar). Calculation of the productivity shock then involves a regression of the Solow residual in levels on its lag. It is straightforward to show that assuming $b(k_t, \theta_t) = b^0 + b^1\theta_t + b^2 k_t$ in equation (20) directly implies the three-factor model:

$$E(r_{t+1}^{iE}) = -b^0\, Cov(\varepsilon_{t+1}, r_{t+1}^{iE}) - b^1 Cov(\theta_t\, \varepsilon_{t+1}, r_{t+1}^{iE}) - b^2 Cov(k_t \varepsilon_{t+1}, r_{t+1}^{iE}). \tag{26}$$

---

betas in all states than larger firms. A reason for why small firms have higher productivity-factor loadings may be because they tend to be of more recent vintage. They thus may rely more on recent technology or be more dependent on new opportunities and looking for niches; each of which are sensitive to productivity/technology fluctuations.

The results for this model are similar to the two-factor case (see Table 2 and Figure 3). Since compared to the three-factor model the two-factor model is more parsimonious, with explanatory power at least as good, we emphasize in the following the two-factor model.

Efficient estimation with fewer test assets

The estimation has employed one-stage GMM estimation mostly because Hansen's optimal weighting matrix is poorly estimated when the cross section of test assets is large relative to the time series. However, because the 25 Fama-French portfolios have an approximate factor structure, much of the cross-sectional dispersion can be captured by only a few portfolios provided they are sufficiently different in value and size characteristics. Accordingly we use as alternative test assets the excess returns of just the four portfolios constituting the HML and SMB factors, available from Kenneth French's website. The advantage is that Hansen's optimal weighting matrix can now be well estimated, allowing for efficient parameter estimation by two-stage GMM.

The results of two-stage GMM estimation for the four test assets are in Table 5, Panel A. The parameter estimates are impressively similar to the 25 test asset case for each of the three model specifications. Focusing on the two-factor conditional model, we find a risk premium of *0.88%* for the productivity factor compared to *0.87%* in the 25 asset case and a risk premium of *-0.62%* for the conditional factor compared to *-0.51%* in the 25 asset case, with similar t-statistics. It is not sensible to consider the R-squares for the four asset case, but the JT statistic for the pricing errors, which adjusts for the degrees of freedom, reflects the improved estimation efficiency, implying a large p-value of *0.55* compared to a p-value of *0.15* in the 25 asset case.

Estimation results for the pre-1992 time period

The original study by Fama and French establishes the value factor as a preeminent influence on asset pricing from data up to 1992. To confirm that our results explain the value effect and are not unduly influenced by the extreme returns of the 1990s we omit this period and consider only the 1964-1992 estimation period. Panel B of Table 5 shows that the results for the reduced sample period are close to the full sample results for each of the three model versions. For instance, the two-factor conditional model implies a risk premium of *0.88%* for the productivity factor, against *0.86%* for the complete sample period, and a risk premium of *-0.41%* for the conditional factor, against *-0.51%* for the complete sample. The adjusted R-square is *51%* compared to *72%* for the full period. The t-statistics are a little lower in accordance with the reduced sample size.

(b) The Importance of the State Variable

The conditional model improves dramatically on the unconditional productivity-based model. The state variable being important however has two alternative implications, not directly related to the cross sectional fit, that we evaluate in this section. First, as argued by Campbell (1996) and others, any state variable explaining cross sectional return differences must be useful in forecasting future returns in a time series context. Second, as argued by Lewellen and Nagel (2005), the quantitative effect of the state variable in explaining cross sectional return differences must be of the same magnitude as the covariance between the risk premium and the asset's beta as in equation (21).

Forecast power of the state variable

Campbell (1996), Ferson and Harvey (1999), Lettau and Ludvigson (2001a), and Santos and Veronesi (2006), following Merton (1973) show that the state variables in their asset pricing models forecast market (excess) returns. Thus to provide additional, independent, evidence on the importance of the cointegrating residual as a state variable we consider its forecasting properties. As Campbell (1996, p.342) notes: "…variables that spuriously explain the crosssection are unlikely to be the same as variables that spuriously forecast the time series." Since the state variables are generally quite persistent, Fama and French (1988) and others argue that the forecast for a *moving sum* of the market return is normally better as this improves the signal-to-noise ratio compared to forecasting a single market return realization. The dividend-price ratio (Fama-French, 1988), the cay variable (Lettau and Ludvigson, 2001a), and the share of labor income (Santos and Versonesi, 2006) all have significant forecasting power for market returns.

We consider these forecast regressions for our state variable, with a few differences. The first is minor: we divide the moving sum of the market return by the number of terms to obtain the moving average. This standardizes the coefficient, making it easier to interpret the results. Second, since we explain the value premium based on the conditional factor, the return on the zero-investment portfolio long on value firms and short on growth firms ("HML") should also be forecastable by the state variable. Third, the theoretical model implies necessarily that the link between market return and forecast variable is quadratic. This issue has been ignored in previous papers but is essential: if the state variable explains unconditional mean return differences it must have an important nonlinear component. From equation (25) it follows that, since the state has zero mean, unconditional mean differences arise only via the quadratic term. Since all assets have positive sensitivity to the conditional factor and since b^1 is positive, the square of the state variable should from equation (25) have a *negative* effect on the future market return – the link between state variable and future market return should be U-shaped. Similarly, since by our explanation of the value effect the zero-investment portfolio loads negatively on the conditional factor, the square of the state variable should have a *positive* effect on the future return difference between value and growth portfolios.

Table 6 demonstrates that indeed the state variable has some forecasting power for market excess returns (S&P 500 returns including dividends, available from Robert Shiller's home page, net of the quarterly T-Bill return), especially for the longer horizons. The R-square for only the state as the explanatory variable is *9*% and *14%* for a one-quarter ahead forecast of the three and four years moving averages of market excess returns, with a significantly negative Newey-West t-statistic for these horizons. When we add the square of the state as an additional forecast variable the coefficient on the square is negative as predicted for all but the three and four-year horizons, but is significant only for the one-quarter horizon.[17] For the HML variable, the explanatory power of the state variable is similar. The R-square is *9*% and *13*% at the three and four-year horizons when the state is the forecast variable; when we add the coefficient for the square of the state variable it is positive as predicted and has a significant Newey-West t-statistic for all but the four-year horizon.[18]

[17] Significance is at the 5% level for a one-tailed test. The adjusted R-squares for the period after 1992 average 50% for the eight-quarter, 12-quarter, and 16-quarter moving average returns, implying that the model does well in explaining the "bubble" of the 1990s. Nevertheless, even for the 1964-1992 period the adjusted R-squares for the eight-quarter, 12-quarter, and 16-quarter moving average market returns are 7-11 percent.

[18] The Hodrick, 1992, type I-B t-statistics are somewhat lower in all cases as reported in Table 6.

Cyclicality of excess returns

Excess returns move counter-cyclically under decreasing relative risk aversion in the endowment economies of Chen (1991) and Campbell and Cochrane (1999): as consumption gets closer to the subsistence or habit level in a recession, risk aversion and required excess returns increase. Empirical confirmation of this pattern is found in Harrison and Zhang (1999) for excess returns and in Tallarini and Zhang (2005) for real returns. They find that predicted market returns (excess and real) are higher during NBER-classified recession periods.

In our production economy, a more complex link between excess returns and the business cycle surfaces. The state variable relates negatively to the productivity risk premium and excess returns (the positive value of b^1) because in a capital-abundant state investment reacts more and consumption varies less in response to a productivity shock. So, as the state variable increases, consumption risk diminishes implying that the productivity risk premium falls unambiguously. The state variable, however, has an ambiguous link to the business cycle as measured by aggregate production. If the state variable rises due to an increase in the capital stock, aggregate production rises and excess returns are countercyclical; if the state variable rises due to a fall in the productivity level, aggregate production falls, ceteris paribus, and excess returns are procyclical. Figure 2 shows that based on the NBER recession classification the state variable often peaks during recessions, suggesting initially that excess returns are mostly procyclical in our model.

To provide more perspective Table 7 provides correlations between our state variable and key variables tied to the business cycle and excess returns. We consider three groups of variables: standard information variables known to forecast returns, business cycle measures, and future market returns. The state variable is uncorrelated with the term premium but has significant negative correlation with the dividend-price ratio and the default risk premium. As is also clear from Figure 2, the state variable has significant positive correlation of *0.45* with a dummy variable that takes the value one during NBER-classified recessions and negative correlation with a continuous business cycle variable (the level of real GDP minus its 12-quarter lagged moving average to adjust for trend). The state variable has negative correlation with one-quarter and 12 quarters ahead cumulative returns, confirming the forecast regression results in Table 6.

Given conventional measures of the business cycle, our state variable is generally countercyclical so that the productivity risk premium is procyclical and, therefore (because productivity risk betas are generally positive), predicted market returns are procyclical. From the perspective of our model, however, aggregate production $f(\theta_t, n_t, k_t)$ is not the best business cycle measure. A better business cycle measure is production plus usable capital, which captures aggregate consumption availability (literally in our model without adjustment costs): $F(\theta_t, n_t, k_t) \equiv f(\theta_t, n_t, k_t) + (1-\delta)k_t$. This is consistent with Black (1990) who finds that aggregate wealth, not production, has a countercyclical link to the risk premium: it is not just current production that matters for consumption opportunities, but also the part of previous production levels saved in the form of capital. The correlation of production plus usable capital with the state variable is a significant *0.58* as shown in Table 7. It follows that excess returns are typically procyclical if we measure the cycle by aggregate production but are countercyclical if we measure the cycle by consumption availability. While the two are identical in a Lucas endowment economy, this is not so in a production economy. Presuming correctness of our model, standing empirical results of countercyclical expected excess returns for the aggregate production measure may be due simply to the significant positive correlation of *0.46* between aggregate production and the consumption availability measure.

The conventional business cycle measures also imply low correlations with future market returns and an unintuitive procyclical default risk premium of *0.14* (which is a countercyclical *-0.31* for the production plus usable capital measure, not shown in Table 7). In addition to the negative correlation with the market risk premium, Table 7 shows that our state variable also has a large significant negative correlation of *-0.59* with the default risk premium. The reason may be, in addition to the productivity risk directly affecting the default risk premium, that an increase in the state represents an increase in tangible capital relative to intangible capital, raising firms' collateral and lowering the default risk premium. Thus, our model provides a more subtle relationship between cycle and excess returns because the relevant state variable does not have a one-dimensional tie to aggregate production.

Magnitude of the conditional effect on returns

To provide more definition for the effect of the state on returns we discretize the state space and consider four different regions for the state variable: more than one standard deviation below the median, between zero and one standard deviations below the median, between zero and one standard deviation above the median and more than one standard deviation above the median. We then first use the Lettau and Ludvigson (2001b) method to compare the productivity risk premium and productivity betas across these four state regimes. The Lettau-Ludvigson method finds for states *i= 1,..,4* the average risk premia in each state and the betas given by equation (24): $\beta^{i\varepsilon}_{s(i)} = \beta^{i0} + \beta^{i1} s(i)$. We can then find the covariance across the four states between risk premium and beta, yielding the conditional premium for each asset. Table 4 provides the results, confirming our earlier discussion that the extent of the (negative) covariance between risk premium and beta explains a significant part of the value premium.

This approach, however, is subject to the Lewellen-Nagel critique. The problem is that the magnitude of the coefficients is based on maximizing the cross sectional fit with no regard to matching the actual covariance between risk premium and betas. Lewellen and Nagel's solution is to estimate betas over short time windows and correlate these with realized risk premia over these short windows. Since we have only quarterly data, this approach is not feasible as is. Instead we provide an approximation by considering a discrete number of four state regimes and estimating betas in addition to obtaining average risk premia for these regimes. By this approach we implicitly set the variation within a state equal to zero. We can decompose the covariance into that from between-state variation and that from within-state variation:

$$Cov(RP,\beta) = E[Cov(RP,\beta) \mid s] + Cov[E(RP \mid s), E(\beta \mid s)] \,. \tag{27}$$

Thus the between-state covariance estimate we obtain is a lower bound to the actual covariance: given a monotonic covariance between risk premium and betas, excluding the first term on the right-hand side decreases the (absolute) magnitude of the total covariance.

To obtain a numerical value for the last term in equation (27) we estimate four productivity betas for each asset, one for each of the state regimes, and find the productivity risk premium for each state regime. We can then find the covariance between risk premium and beta for all 25 assets. Impressively, the covariance is negative in all cases. Table 4 shows that, as for the LL approach, the risk premium varies directly by state going from *0.98%* in the worst state (*s* is more than one standard deviation below the median) to *0.18%* in the best state (*s* is more than one standard deviation above the median). Although there is no theoretical reason for the risk premium to be positive, we find no evidence of a systematically negative risk premium.

Table 4 also shows quite clearly that betas rise mostly monotonically from the worst state to the best state and that this effect is more pronounced for growth firms than for value firms. Most importantly, when we calculate the (lower bound of the) quantitative value for the covariance for value firms and growth firms in each size class, this covariance in Panel B of Table 4 is always negative and more so for growth firms, explaining a positive value premium for each size class that is on average around *56%* of the value premium in each size class estimated in the model as shown in Panel D of Table 3 and around *44%* of the observed value premium as shown in Panel A of Table 3. The shortfall can plausibly be attributed to the fact that our four-state approximation ignores the within-state covariation – the first term on the right-hand side of equation (27). We conclude that the covariance between productivity risk premium and productivity betas is quantitatively strong so that the Lewellen-Nagel critique appears not to be a problem in our production-based context.

(c) Model Comparison

Results for unconditional models

Table 8 reports the results for the unconditional version of our model and for three widely cited unconditional models: the CCAPM, the CAPM and the Cochrane (1996) Investment-based CAPM. According to the *JT* value , all models are rejected (possibly due to poor estimation of the spectral density matrix). Figure 5 displays the alphas and the cross sectional fit for each model.

Figure 5 and Table 8 show that the cross sectional fits of the productivity-based model and the investment-based model are similar, around *20%*, and significantly higher than for the CCAPM and CAPM which perform poorly in explaining the cross section of the 25 size and book-to-market sorted assets with adjusted R-square below *1%*. The risk premia are not statistically significant for any of the four models if we consider the GMM t-statistics. The quarterly alphas are above *2%* (*8%* on an annual basis) and statistically significant for the CCAPM and CAPM. For the Investment-based model the alpha is *1.3%* but not statistically significant, and for the production-based model the alpha is a statistically insignificant *0.8%*.

Results for the conditional models and the Fama-French model

We now consider conditional models and the Fama-French 3-factor model. The Fama-French model can be interpreted as a version of the Merton model. Two recent conditional consumption-based models are included: the Lettau and Ludvigson (2001) "cay" model and the Campbell and Cochrane (1999) habit persistence model. In addition we consider our productivity-based two-factor conditional model using equation (23).

Table 9 shows that our model has the lowest *JT* statistic, substantially lower than that of any of the other models, including the Fama-French model. Figure 6 provides the adjusted R-square which is *71.5%* for our model compared to *77.6%* for the Fama-French model. The Lettau-Ludvigson and Campbell-Cochrane models have R-squares of *46.1%* and *40.4%* respectively. The alphas for the productivity-based conditional model and cay model are statistically insignificant and relatively low at *0.48%* and *0.15%,* respectively. The alphas for the Fama-French and Campbell-Cochrane models are statistically and economically significant, however, at *3.92%* and *2.07%,* respectively.

(d) Model Evaluation

A recent paper by Lewellen, Nagel, and Shanken (2006) argues that the cross sectional fit of the 25 Fama-French portfolios sorted by size and value presents a rather low hurdle for the evaluation of an asset pricing model. They contend that when portfolios have a strong factor structure, as is the case for the 25 Fama-French portfolios, the pricing errors are idiosyncratic and not likely to be correlated with the factors of a particular asset pricing model; but, in that case, even a weak correlation between the model factors and those generating the test portfolios implies that the model betas are linearly related to the "true" betas, automatically generating a high R-square.

In the following we discuss the prescriptions suggested by Lewellen, Nagel, and Shanken (LNS) to provide alternative ways to evaluate asset pricing models. First LNS suggest adding Fama and French's 30 industry portfolios to the 25 size and value portfolios and using these assets jointly to test a model in order to relax the tight factor structure of the size-value portfolios. Table 10 provides the results for the three variants of our model and the conditional models. In each case the R-square deteriorates significantly. Both of our conditional models hold up relatively well, with R-squares for our 2-factor and 3-factor models of *22.8%* and *33.1%,* respectively, compared to *31.5%* for the FF model and much lower for the other models.

Isolating the performance for the 30 industry portfolios, we present the root mean square error of these portfolios implied by the parameter estimates for the 55 test assets. These are relatively similar for all models, varying from *0.75* percent for the Campbell-Cochrane model to *0.81* percent for the Fama-French model. Along this dimension, all three of our productivity-based models outperform the Lettau-Ludvigson and Fama-French models but underperform the Campbell-Cochrane model. The relatively strong performance of the unconditional productivity-based model here is likely due to the fact that it does worse than the conditional models in explaining the 25 size and value-sorted portfolios so that the parameter estimation can emphasize fitting the 30 industry portfolio returns without giving up much explanatory power. A possible explanation for the poor performance of all models in explaining the average industry returns is the occurrence of persistent idiosyncratic industry-specific shocks: with hindsight certain industries decline while others emerge. If the trends are not discernible in advance, average returns cannot fully represent risk factors.

Second LNS advocate checking whether estimated slopes and intercept are reasonable. We find that the alpha in our 2-factor model is economically reasonable, implying a quarterly mispricing of zero-beta assets of *0.48%,* except for the *cay* model lower than the other models. In addition the alpha is statistically insignificantly different from zero, even when the 30 industry portfolios are added as test assets. Note that for the FF model the alpha is economically as well as statistically significant: it has a quarterly alpha exceeding *3%*. We have discussed the reasonability of the unconditional and conditional factor premium estimates previously, arguing in particular that the size of the conditional factor risk premium is related to the magnitude of covariance between beta and risk premium. The coefficients decrease in magnitude when the 30 industry portfolios are added but this is in line with the overall degradation in performance of the model.

Third LNS propose to include the model's factors among the test assets. Since our factors are not returns we use a slightly different approach and evaluate our 2-factor conditional model's performance in pricing the investment return for the representative firm. We find that the model implies a production function parameter of *0.24* which is relatively close to the typically assumed value of *0.33*.

As final prescriptions LNS suggest reporting confidence intervals for the cross sectional R-square and for the squared pricing errors measure. We present the 90% confidence intervals for the R-square and for the *JT* statistic in Table 10. The confidence intervals for both are based on the distribution of the statistic, for a particular group of test assets, obtained in 5,000 draws of the appropriate number of multivariate normally distributed factors. The interval reported contains the 90% of R-squares around the median. Table 10 shows that only the R-squares of our 2-factor model and the FF model exceed the 90% confidence interval. When the industry test assets are added none of the models have R-squares outside of the interval. Results for the JT statistic are similar: except for our 2-factor model, no models have JT statistics outside of the 90% confidence interval based on random factors. For the 55 assets case, our 2-factor models has a JT statistic below the 90% confidence interval, indicating that the pricing errors are significantly smaller than for a random factor model.

In addition to the LNS prescriptions we have considered several other diagnostics that illustrate the reasonability of our productivity-based model. To summarize, we have shown not only that the magnitude of the conditional factor risk premium is sensible but more specifically that the risk premium decreases state by state as the state improves over the four regimes we distinguish (Table 4). The state variable significantly forecasts future excess returns on the value factor and the market return in the direction predicted by the theory. And, most importantly, the factors are exactly those implied by theory for general specifications of the standard RBC model.

5. Conclusion

We study the cross section of asset returns from the production side of the economy and argue that asset returns are determined by a one factor model with one conditioning variable. The factor is the aggregate productivity shock ε_{t+1}, the impact of which is conditional on the state of the economy s_t. In the context of our model—a standard RBC model—the state of the economy as it pertains to asset prices is fully characterized by the relative scarcity of capital: the level of the existing capital stock (tangible capital) in comparison to productivity (intangible capital).

In general terms, we replace a consumption-based pricing kernel with a production-based kernel, employing a stochastic link between the marginal rate of substitution and the marginal rate of transformation according to which $u_c(c_{t+1}, x_{t+1}) = m(\varepsilon_{t+1}, s_t)$, where x evokes a variety of variables, each potentially affecting the marginal utility of consumption,

The empirical results strongly support our production-based approach and model compared to existing theoretical models in the literature, and is competitive empirically with the Fama-French three-factor model. Thus, the cross section of asset prices provides support for the canonical RBC model from a different perspective. Furthermore, it appears that the speculated improved measurement from using production variables instead of consumption variables indeed provides a better characterization of the cross section of asset prices.

The explanations for the mean returns of the challenging 25 Fama-French test assets sorted by size and value is that small firms have higher mean returns than large firms mostly because their average sensitivities to productivity risk are higher. Value firms have higher mean returns than growth firms, in spite of having approximately the same average sensitivity to productivity risk, because the risk premiums and risk sensitivities vary with the capital stock and productivity level:

As the state variable (capital stock relative to productivity level) increases, the risk premium falls while risk sensitivities to the productivity shock increase. But compared to value firms the risk sensitivities of the growth firms increase more rapidly with the state so that growth firms have their highest risk when the productivity risk premium is low (or even negative), implying lower returns on average. The reason that growth firms are more sensitive to changes in the state may be that the effective price of investment varies inversely with the state. Thus, growth firm values vary more dramatically with productivity shocks in a capital abundant state, as positive shocks are enhanced with low investment costs enabling these firms to realize their growth potential cheaply.

The consumption-based approach has had difficulty explaining the size of the equity premium, mean return differences across asset classes, and the low covariance between betas and risk premia. Our production-based approach deals successfully with the second and third issues, respectively, by explaining a substantial fraction of cross sectional return variation from productivity shocks and a conditional variable, and by generating a covariance between productivity shock betas and risk premium across state regimes sufficient to explain quantitative differences in returns across asset classes.

While the model is effective in explaining cross-sectional differences, we do not address the first issue, the equity premium puzzle, and, more generally, do not ask whether the overall level and volatility of observed returns can be generated by a model of this kind. This requires a specific functional form for the marginal value of capital function, which in turn, requires a full specification of the general equilibrium model. But one of the advantages of our approach has exactly been the ability to avoid making such specific assumptions: our results hold independent of functional forms, relying simply on the key variables underlying basic RBC models.

Nevertheless, under the assumption that the CCAPM has generally failed to meet the three challenges because of the incorrect specification of the *x* factor affecting marginal utility, we see no reason why a proper functional form for the marginal value of capital function would not resolve the equity premium puzzle. For one, fitting the aggregate investment return provides a reasonable value for the capital share in production for the basic Cobb-Douglas production function.

It may be that quantitatively the factor sensitivities needed to explain the cross sectional return variations are unrealistically high (or low for that matter). In this case, consider that we present the simplest RBC model to emphasize that our cross sectional results are consistent with it. It is possible to change the investment function to allow adjustment costs, changing the form of the marginal value of capital function without changing the variables – the productivity factor or the state variable. This would allow the market price of capital to vary, generating return volatility supplementing that due to productivity shocks. Finding an investment cost function (with or without adjustment costs) in the context of a fully calibrated general equilibrium model that implies a realistic equity premium and return volatilities presents an appealing direction for future research.

References

Abel, Andrew B, 1990, "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review* 80, 38-42.

Bahk, Byong-Hong, and Michael Gort, 1993, "Decomposing Learning by Doing in New Plants," *Journal of Political Economy*, 101, 561-83.

Balvers, Ronald J., Thomas F. Cosimano, and Bill McDonald, 1990, "Predicting Stock Returns in an Efficient Market," *Journal of Finance* 45, 1109-1128.

Balvers, Ronald J. and Dayong Huang, 2004, "Money and the (C)CAPM: Theory and Evaluation," Working Paper, West Virginia University.

Basu, Susanto, John Fernald, and Matthew Shapiro, 2000, "Productivity Growth in the 1990s: Technology, Utilization or Adjustment?" NBER Working Paper 5865.

Black, Fischer, 1990, "Mean Reversion and Consumption Smoothing," *Review of Financial Studies* 3, 107–114.

Boldrin, Michele, Lawrence Christiano, and Jonas Fisher, 2001, "Habit persistence, asset returns, and the business cycle," *American Economic Review* 91, 149–166.

Breeden, Douglas T., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics* 7, 265-296.

Brock, William A., 1982, "Asset Prices in an Exchange Economy," in John J. McCall, ed.: *The Economics of Information and Uncertainty*, University of Chicago Press, Chicago.

Cecchetti, Stephen, Pok-Sang Lam, and Nelson Mark, 1990, "Mean Reversion in Equilibrium Asset Prices," *American Economic Review* 80, 398-418.

Campbell, John Y., 1993, "Intertemporal Asset Pricing Without Consumption Data," *American Economic Review* 83, 487-512.

Campbell, John Y., 1996, "Understanding Risk and Return," *Journal of Political Economy* 104, 298–345.

Campbell, John Y., and John H. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy* 107, 205-251.

Chen, Nai-fu, 1991, "Financial Investment Opportunities and the Macroeconomy," *Journal of Finance* 46, 1467-1484.

Cochrane, John H., 1991, "Production-Based Asset Pricing and the Link between Stock Returns and Economics Fluctuation," *Journal of Finance* 46, 209-237.

Cochrane, John H., 1996, "A Cross-Sectional Test of an Investment-Based Asset Pricing Model," *Journal of Political Economy* 104, 572-621.

Cochrane, John H., 2001, *Asset Pricing*, Princeton University Press, New Jersey.

Cochrane, John H., 2005, "Financial markets and the real economy," Working Paper, University of Chicago.

DeJong, David. N. and Beth.F. Ingram, 2001, The Cyclical Behavior of Skill Acquisition." *Review of Economic Dynamics* 4, 536-561.

Fama, Eugene F., and Kenneth R. French, 1988, "Permanent and Temporary Components of Stock Prices, *Journal of Political Economy* 96, 246-273.

Fama, Eugene F., and Kenneth R. French, 1996, "Multifactor explanations of asset pricing anomalies," *Journal of Finance* 51, 55-84.

Fama, E. F. and J. D. MacBeth, 1973, "Risk Return and Equilibrium: Empirical Tests," *Journal of Political Economy* 71, 607-636.

Ferson E. Wayne, and Campbell R. Harvey, 1999, "Conditioning Variables and the Crosssection of Stock Returns," *Journal of Finance* 54, 1325-1360.

Gomes, Joao, Leonid Kogan, and Lu Zhang, 2003, "Equilibrium Cross Section of Returns," *Journal of Political Economy* 111, 693-732.

Hansen, Lars Peter, 1982, "Large Sample Properties of Generalized Method of Moments Estimation," *Econometrica* 50, 1029-1054.

Harrison, Paul, and Harold H. Zhang, 1999, "An Investigation of the Risk and Return Relation at Long Horizons," *Review of Economics and Statistics* 81, 300-408.

Hodrick, Robert and X. Zhang (2001). "Evaluating the Specification Errors of Asset Pricing Models," *Journal of Financial Economics* 62, 327-376.

Jaganathan, Ravi and Zhenyu Wang, 1998, "An Asymptotic Theory for Estimating Beta-Pricing Models Using Cross-Sectional Regressions," *Journal of Finance* 53, 1285-1309.

Jaganathan, Ravi and Zhenyu Wang, 2002, "Empirical Evaluation of Asset Pricing Models: A Comparison of the SDF and Beta Method," *Journal of Finance* 57, 2337-2367.

Jermann, Urban J., 1998, "Asset Pricing in Production Economies," *Journal of Monetary Economics* 41, 257-275.

Jermann, Urban J, 2005, "The Equity Premium Implied by Production," Working Paper, University of Pennsylvania, March 2005.

Jorgenson, Dale W., and Kevin J. Stiroh, 1999, Information Technology and Growth," *American Economic Review* 89, 109-115.

Kim, Youn H., 2003, "Intertemporal Production and Asset Pricing: A Duality Approach," *Oxford Economic Papers* 55, 344-379.

King, Robert G. and Sergio T. Rebelo, 2000, "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics*, vol. IB, John B. Taylor and Michael Woodford editors, Chapter 14.

King, Robert G., Charles I. Plosser, James H. Stock, and Mark W. Watson, 1991, "Stochastic Trends and Economic Fluctuations," *American Economic Review* 81, 819-840.

Lettau, Martin., and Sydney C. Ludvigson, 2001a, "Consumption, Aggregate Wealth, and Expected Stock Returns," *Journal of Finance* 56, 815-849.

Lettau, M. and Sydney C. Ludvigson, 2001b, "Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia are Time-Varying," *Journal of Political Economy* 109, 1238-1287.

Lewellen, Jonathan, and Stefan Nagel, 2004, "The conditional CAPM does not explain asset-pricing anomalies," forthcoming *Journal of Financial Economics*.

Lewellen, Jonathan, Stefan Nagel, and Jay Shanken, 2006, "A Skeptical Appraisal of Asset-Pricing Tests," working paper.

Li, Qing, Maria Vassalou, and Yuhang Xing, 2004, "Sector Investment Growth Rates and The Cross–Section of Equity Returns," forthcoming *Journal of Business.*

Liew, Jimmy, and Maria Vassalou, 2000, "Can Book-to-Market, Size and Momentum be Risk Factors that Predict Economic Growth?" *Journal of Financial Economics* 57, 221-245.

Lucas, Robert E. Jr., 1978, "Asset Prices in an Exchange Economy," *Econometrica* 46, 1429-1445.

Mankiw, N. Gregory, and Matthew D. Shapiro, 1986, "Risk and Return: Consumption Beta versus Market Beta" *Review of Economics and Statistics* 68, 452–459.

Mehra, Rajnish, and Edward Prescott, 1985, "The Equity Premium Puzzle," *Journal of Monetary Economics* 15, 145-161.

Merton, Robert C., 1973, "An Intertemporal Capital Asset Pricing Model," *Econometrica* 41, 867-887.

Parker, Jonathan A. and Christian Julliard, 2005, "Consumption Risk and the Cross Section of Expected Returns," *Journal of Political Economy* 113, 185-222.

Peng, Yajun and Hany Shawky, 1999, "Stochastic Discount Rates, Productivity Shocks and Capital Asset Pricing," *Review of Quantitative Finance and Accounting* 12, 21-34.

Petkova, Ralitsa, and Lu Zhang, 2005, "Is Value Riskier Than Growth?" forthcoming *Journal of Financial Economics*.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2003, "Housing, Consumption, and Asset Pricing," forthcoming *Journal of Financial Economics*.

Santos, Tano, and Pietro Veronesi, 2006, "Labor Income and Predictable Stock Returns," *Review of Financial Studies* 19, 1-44.

Shanken, Jay, 1992, "On the Estimation of Beta-Pricing Models," *Review of Financial Studies* 5, 1-33.

Sims, Christopher A., James H. Stock, and Mark W. Watson, 1990, "Inference in Linear Time Series Models with Some Unit Roots." *Econometrica* 58, 113-144.

Tallarini, Thomas D. and Harold H. Zhang, 2005, "External Habit and the Cyclicality of Expected Stock Returns," *Journal of Business* 78, 1023-1048.

Vassalou, Maria and Kodjo Apedjinou, 2003, "Corporate Innovation and its Effects on Equity Returns," manuscript, Columbia University.

Vassalou, Maria, 2003, "News Related to Future GDP Growth as a Risk Factor in Equity Returns," *Journal of Financial Economics* 68, 47-73.

Yogo, Motohiro, 2003, "A Consumption-Based Explanation of Expected Stock Returns," forthcoming *Journal of Finance*.

Zhang, Lu, 2005, "The Value Premium," *Journal of Finance* 60, 67-103

Table 1. Summary statistics for the growth rate of per capita capital *k*, productivity θ, the productivity shock ε and the cointegrating residual *s*. The data start at 1964Q1 and end at 2004Q4. Panel B shows the results of the ADF unit root tests with various lags for *k* and θ and including a constant and a time trend. Panel C reports the cointegrating vector between *k* and θ and the ADF test results for cointegration of *k* and θ with various lags and including a constant but not a time trend. The Akaike Information Criterion suggests that 2 lags are optimal. Panel D tests cointegration between *k* and θ using Johansen's method. The likelihood ratio test suggests that the optimal lag length is 3. The trace and eigenvalue test statistics show that the null of no cointegrating relation can be rejected. Panel E reports autocorrelations of ε and *s*. Panel F reports regression results for whitening the productivity shock. Panel G checks the existence of autocorrelation or a time trend in *s* and ε. Panel H shows the regression of private investment growth on the productivity shock, state variable, and their interaction.

Panel A	Δ*k*	Δθ	ε	*s*
Mean	0.73%	0.19%	0.00%	0.00
Std.	0.52%	0.78%	0.77%	0.56
ρ (Δ*k*, Δθ)	-0.22			

Panel B	Critical Values: -3.46 (1%), -2.91 (5%), -2.59 (10%)			
Lags	1	2	3	4
k	6.95	3.94	3.08	2.75
θ	-0.07	-0.16	-0.39	-0.17

Panel C	Critical Values: -4.02 (1%), -3.40 (5%), -3.09 (10%)			
s	-2.47	-2.74	-2.59	-2.48
	Cointegrating Vector: (1,-0.067)			

Panel D	Crit. Val: 19.93 (1%), 15.49 (5%)		Crit. Val: 18.52 (1%), 14.26 (5%)	
	Trace Test:	25.15	Eigen Test:	18.65

Panel E	1	2	3	4
ε	0.089	0.086	-0.021	-0.034
s	0.945	0.875	0.797	0.722

Panel F	Constant	θ(t-1)	*k*(t-1)	R-square
ε(t)	0.054	-0.020	0.287	3.10%
t-stat	*2.588*	*-2.608*	*2.733*	

Panel G	Constant	s(t-1)	ε(t-1)	time
s(t)	0.000	0.942		0.000
t-stat	*-0.779*	*36.201*		*0.884*
ε(t)	0.000		0.083	0.000
t-stat	*-0.088*		*1.051*	*0.117*

Panel H	Constant	s(t-1)	ε(t)	s(t-1) ε(t)
ΔI(t)	0.007	-0.007	0.981	1.023
t-stat	*3.760*	*-1.995*	*3.823*	*2.130*

Table 2. GMM one-stage estimates for the productivity based models. We report the pricing kernel loadings on the systematic factors (the b^i parameters), the implied risk premiums and their traditional (Fama-MacBeth, 1973), Shanken (1992) corrected, and autocorrelation consistent t-values, Hansen's *JT* statistics, and implied R-squares. PROD is the unconditional productivity model; PROD-*s* uses the cointegrating residual *s* between *k* and θ as the conditioning variable; and PROD-*kθ* includes the two state variables *k* and θ each adjusted for a linear time trend. The data are from 1964Q1 to 2004Q4.

PROD	Constant	ε				
b	0.991	-56.285			JT	95.739
t-statistics	*26.971*	*-1.046*			*p-value*	*0.000*
Risk Premium %	0.772	0.356				
t-statistics	*0.947*	*1.357*			R-square	18.18%
Shanken-t	*0.862*	*1.240*				
GMM-t	*0.780*	*1.139*				
PROD-*s*	Constant	ε	ε.*s*			
b	0.995	-224.4	438.36		JT	28.859
t-statistics	*5.808*	*-1.494*	*1.668*		*p-value*	*0.149*
Risk Premium %	0.475	0.869	-0.510			
t-statistics	*0.583*	*3.120*	*-2.620*		R-square	71.53%
Shanken-t	*0.257*	*1.402*	*-1.168*			
GMM-t	*0.353*	*1.368*	*-1.310*			
PROD-*kθ*	Constant	ε	ε.θ	ε.*k*		
b	1.010	-227.3	-14.76	45.04	JT	24.500
t-statistics	*6.856*	*-1.786*	*-0.331*	*1.670*	*p-value*	*0.270*
Risk Premium %	1.532	0.867	0.301	-1.700		
t-statistics	*1.824*	*3.380*	*0.874*	*-1.760*	R-square	68.74%
Shanken-t	*0.939*	*1.777*	*0.463*	*-0.925*		
GMM-t	*1.108*	*1.797*	*0.502*	*-0.937*		

Table 3. Returns for the 25 portfolios sorted by size and book-to-market ratio, of which the returns of the 10 portfolios in the extreme value and growth classes are displayed. In all panels the results are shown for value and growth and value minus growth (HML) portfolios for each size quintile. Panel A displays the actual mean returns. Panel B shows the predicted returns. Panels C and D provide the unconditional, $a - b^0\, Cov(\varepsilon_{t+1}, r^{iE}_{t+1})$, and conditional, $-b^1\, Cov(s_t \varepsilon_{t+1}, r^{iE}_{t+1})$, excess return components.

	Return Decomposition					
	Panel A: Actual Return			**Panel B: Predicted Return**		
	Value	Growth	HML	Value	Growth	HML
Small	3.89%	1.15%	2.74%	2.80%	1.70%	1.10%
Semi-Small	3.36%	1.55%	1.81%	2.68%	1.39%	1.29%
Medium	3.25%	1.54%	1.72%	2.62%	1.15%	1.47%
Semi-Big	2.75%	1.80%	0.95%	2.54%	0.96%	1.59%
Big	1.79%	1.30%	0.49%	1.71%	0.44%	1.27%
	Panel C: Unconditional Return Component			**Panel D: Conditional Return Component**		
	Value	Growth	HML	Value	Growth	HML
Small	4.54%	4.63%	-0.09%	-1.74%	-2.93%	1.19%
Semi-Small	3.79%	3.91%	-0.12%	-1.11%	-2.52%	1.41%
Medium	3.67%	3.38%	0.29%	-1.05%	-2.23%	1.18%
Semi-Big	3.40%	2.96%	0.45%	-0.86%	-2.00%	1.14%
Big	2.23%	2.18%	0.05%	-0.52%	-1.74%	1.22%

Table 4. The productivity risk premium and productivity betas by state. We distinguish four states: a high capital scarcity state (State 1: *s* more than one standard deviation below the median), moderate capital scarcity (State 2: *s* below the median), moderate capital abundance (State 3: *s* above the median) and a high capital abundance state (State 4: *s* more than one standard deviation above the median). Results are reported for the growth and value portfolio productivity betas for each state and for their difference. In addition the table provides the covariance of the portfolio's productivity betas with the productivity risk premium. The LL approach (Panel A) computes the betas for each state using equation (24) while the LN approach (Panel B) computes the betas using two pass regression for each state separately.

	State 1	**State 2**	**State 3**	**State 4**	**State 1**	**State 2**	**State 3**	**State 4**
Risk Premium	0.98%	0.32%	0.20%	0.18%	0.98%	0.32%	0.20%	0.18%
	Panel A: LL APPROACH				**Panel B**: LN APPROACH			
Growth portfolio Betas								
Small	0.74	3.31	6.46	10.39	0.06	5.39	4.49	11.54
Semi-Small	0.54	2.76	5.48	8.87	-1.76	4.14	4.31	10.05
Medium	0.39	2.35	4.76	7.75	-0.68	3.70	3.69	8.59
Semi-Big	0.26	2.03	4.18	6.87	-0.30	3.19	3.08	7.84
Big	-0.22	1.31	3.18	5.52	1.86	1.44	2.60	6.50
Value portfolio Betas								
Small	2.49	4.02	5.90	8.23	2.93	5.30	4.35	9.17
Semi-Small	2.62	3.60	4.79	6.29	2.21	4.60	3.44	7.50
Medium	2.56	3.49	4.63	6.05	5.10	3.51	3.85	7.04
Semi-Big	2.57	3.33	4.25	5.41	1.27	3.99	2.76	7.17
Big	1.74	2.20	2.76	3.46	2.07	2.97	0.76	5.80
HML portfolio Betas								
Small	1.75	0.71	-0.57	-2.16	2.88	-0.09	-0.14	-2.37
Semi-Small	2.08	0.84	-0.68	-2.58	3.97	0.46	-0.87	-2.56
Medium	2.18	1.14	-0.13	-1.71	5.77	-0.19	0.16	-1.55
Semi-Big	2.31	1.30	0.07	-1.47	1.57	0.80	-0.32	-0.67
Big	1.97	0.89	-0.42	-2.06	0.21	1.54	-1.84	-0.69
Cov(Beta,Risk Premium)	Value	Growth	HML		Value	Growth	HML	
Small	-0.63%	-1.25%	0.62%		-0.68%	-1.42%	0.74%	
Semi-Small	-0.42%	-1.18%	0.77%		-0.59%	-1.59%	0.99%	
Medium	-0.01%	-1.01%	1.00%		-0.01%	-1.21%	1.19%	
Semi-Big	-0.33%	-0.87%	0.54%		-0.67%	-1.02%	0.34%	
Big	-0.08%	-0.44%	0.36%		-0.24%	-0.41%	0.17%	

Table 5. Panel A presents the two stage GMM estimation results for the three productivity based models. The four test assets are the value firm, growth firm, small firm, and large firm portfolio returns available from Kenneth French's website. Panel B displays the one stage estimation results for the 1964-1992 sample period for the 25 value and size sorted portfolio returns. PROD is the unconditional productivity model; PROD-*s* uses the cointegrating residual *s* between *k* and θ as the conditioning variable; and PROD-*kθ* includes the two state variables *k* and θ each adjusted for a linear time trend. The data are from 1964Q1 to 2004Q4.

	Panel A: Two Stage GMM					Panel B: 64-92 Sample Period					
PROD	Constant	ε		JT	10.95	Constant	ε			JT	69.14
b	1.00	-133.95		p-value	0.00	1.01	-50.40			p-value	0.00
t-statistics	11.62	-1.93				18.08	-0.97			R-square	24.42%
Risk Premium %	0.41	0.40				0.29	0.38				
t-statistics	0.48	1.47				0.30	1.28				
Shanken-t	0.42	1.31				0.28	1.17				
GMM-t	0.36	1.18				0.26	1.02				
PROD-*s*	Constant	ε	ε.*s*	JT	0.35	Constant	ε	ε.*s*		JT	26.29
b	0.98	-303.82	600.01	p-value	0.55	1.04	-162.59	332.01		p-value	0.24
t-statistics	4.11	-1.68	1.44			6.09	-1.29	1.30		R-square	51.32%
Risk Premium %	0.98	0.88	-0.62			0.09	0.88	-0.41			
t-statistics	1.13	3.00	-2.52			0.09	2.45	-2.23			
Shanken-t	0.45	1.22	-1.01			0.05	1.29	-1.17			
GMM-t	0.51	1.15	-1.13			0.06	1.31	-1.11			
PROD-*kθ*	Constant	ε	ε.θ	ε.*k*		Constant	ε	ε.θ	ε.*k*	JT	21.34
b	1.02	-293.46	-17.18	62.13		1.18	-201.91	-0.73	39.82	p-value	0.44
t-statistics	5.35	-1.72	-0.09	1.40		6.25	-1.57	-0.02	1.57	R-square	68.64%
Risk Premium %	1.69	1.08	0.00	-0.02		0.47	0.92	0.002	-0.01		
t-statistics	1.97	4.02	0.43	-2.35		0.50	2.64	0.42	-1.32		
Shanken-t	0.82	1.70	0.18	-1.00		0.28	1.48	0.24	-0.75		
GMM-t	0.93	2.01	0.19	-1.05		0.33	1.51	0.27	-0.71		

Table 6. Predicting market excess returns (MKT) and the value premium (HML) using *s* and s^2. We report the coefficients, Newey-West t-values and Hodrick (1992) type I-B t-values, and the adjusted R-square. Panel A uses *s* as the only forecasting variable. Panel B uses both *s* and s^2 as forecast variables. * denotes that estimates are significant at the 5% level of significance for a one-tailed test based on the Newey-West t-statistic. MKT represents excess market returns based on the S&P 500 Composite Index (including dividend, available from Robert Shiller's home page) and subtracting the T-Bill rate available from Kenneth French's home page. HML is the value factor from Kenneth French's home page.

		MKT				**HML**			
		Panel A: *s*							
Quarters		Estimates (times 100)	*Newey-West*	*Hodrick*	R-square	Estimates (times 100)	*Newey-West*	*Hodrick*	R-square
1	*s*	-1.83	-1.56	-1.31	1.27%	0.34	0.31	0.31	-0.53%
4	*s*	-1.01	-1.13	-0.83	1.42%	0.27	0.32	0.29	-0.44%
8	*s*	-1.02	-1.53	-0.97	3.37%	0.79	1.40	0.97	2.76%
12	*s*	-1.43*	-3.17	-1.39	9.28%	1.11*	2.56	1.39	8.96%
16	*s*	-1.76*	-3.55	-1.60	14.09%	1.33*	3.04	1.58	13.38%
		Panel B: *s* and s^2							
Quarters		Estimates (times 100)	*Newey-West*	*Hodrick*	R-square	Estimates (times 100)	*Newey-West*	*Hodrick*	R-square
1	*s*	-1.01	-0.96	-0.88	2.95%	-0.29	-0.29	-0.26	0.70%
	s^2	-2.92*	-2.21	-1.52		2.21*	2.14	1.59	
4	*s*	-0.68	-0.84	-0.64	2.03%	-0.16	-0.20	-0.16	1.69%
	s^2	-1.14	-1.00	-0.78		1.52*	1.90	1.42	
8	*s*	-0.90	-1.63	-0.94	3.04%	0.46	0.77	0.51	5.58%
	s^2	-0.41	-0.45	-0.33		1.16*	1.88	1.28	
12	*s*	-1.45*	-3.29	-1.53	8.70%	0.92*	2.12	1.10	11.15%
	s^2	0.10	0.16	0.09		0.85*	1.66	1.01	
16	*s*	-1.79*	-3.82	-1.69	14.62%	1.28*	3.38	1.54	16.11%
	s^2	0.76	1.00	0.50		1.01	1.58	0.98	

Table 7. Correlations of our state variable *s*, detrended output (the level of real GDP minus its 12 quarter lagged moving average) *y* and 12 quarter forward moving average market excess returns *MKT12Q* with commonly cited information variables (dividend-price ratio *Div*, the default premium *Def*, and the term premium *Term*), business cycle variables (dummy for NBER classified recessions periods *NBER*, detrended output *y*, and detrended real output plus undepreciated capital, the level of real GDP plus undepreciated capital minus its 12 quarter lagged moving average, $y+(1-\delta)\,k$), and market risk premium measures (the 1-quarter and 12-quarter forward moving average market excess returns *MKT1Q* and *MKT12Q*). * denotes that correlations are significant at the 5% level of significance for a two-tailed test.

	Div	*Def*	*Term*	*NBER*	*y*	$y+(1-\delta)\,k$	*MKT1Q*	*MKT12Q*
s	-0.20*	-0.59*	-0.03	0.45*	-0.24*	0.58*	-0.15	-0.35*
y	-0.38*	0.14	-0.45*	-0.56*	1.00*	0.46*	-0.07	-0.17*
MKT12Q	0.62*	0.25*	0.33*	0.16*	-0.16*	-0.46*	0.30*	1.00*

Table 8. GMM one stage estimation for unconditional models. Models are PROD, CCAPM, CAPM and INV which are, respectively, the unconditional productivity model, the consumption-based CAPM, the CAPM and Cochrane's (1996) investment-based asset pricing model.

PROD	Constant	ε			
b	0.991	-56.285		JT	95.739
t-statistics	26.971	-1.046		*p-value*	*0.000*
Risk Premium %	0.772	0.356			
t-statistics	0.947	1.357		R-square	18.18%
Shanken-t	0.862	1.240			
GMM-t	0.780	1.139			
CCAPM	Constant	ΔC			
b	1.031	-9.655		JT	113.301
t-statistics	2.000	-0.104		*p-value*	*0.000*
Risk Premium %	2.127	0.036			
t-statistics	2.734	0.204		R-square	0.44%
Shanken-t	2.733	0.204			
GMM-t	2.429	0.206			
CAPM	Constant	Mkt			
b	0.964	0.549		JT	124.037
t-statistics	31.556	0.375		*p-value*	*0.000*
Risk Premium %	2.680	-0.300			
t-statistics	2.885	-0.264		R-square	0.79%
Shanken-t	2.892	-0.265			
GMM-t	2.538	-0.265			
INV	Constant	ΔInv	ΔNon-Inv		
b	1.474	-51.276	-1.797	JT	70.374
t-statistics	4.138	-1.659	-0.276	*p-value*	*0.000*
Risk Premium %	1.300	2.814	1.687		
t-statistics	1.594	2.903	1.399	R-square	20.94%
Shanken-t	1.014	1.866	0.916		
GMM-t	1.089	1.738	1.175		

Table 9. GMM one stage estimation for conditional models. Models are PROD-*s*, FF3, CCAPM-CAY and CCAPM-C which are, respectively: the 2-factor conditional productivity model, the Fama and French (1996) 3-factor model, the Lettau and Ludvigson (2001) conditional (*cay* as conditioning variable) Consumption CAPM, and the Campbell and Cochrane (1999) Consumption CAPM with habit persistence.

PROD-*s*	Constant	ε	ε.*s*			
b	0.995	-224.4	438.36		JT	28.859
t-statistics	5.808	-1.494	1.668		*p-value*	*0.149*
Risk Premium %	0.475	0.869	-0.510			
t-statistics	0.583	3.120	-2.620		R-square	71.53%
Shanken-t	0.257	1.402	-1.168			
GMM-t	0.353	1.368	-1.310			
FF3	Constant	Mkt	Smb	Hml		
b	0.952	5.443	-5.939	-1.132	JT	81.904
t-statistics	11.870	2.179	-2.560	-0.503	*p-value*	*0.000*
Risk Premium %	3.923	-2.416	0.811	1.393		
t-statistics	3.327	-1.758	1.640	2.633	R-square	77.57%
Shanken-t	3.069	-1.652	1.634	2.601		
GMM-t	2.979	-1.837	1.513	2.397		
CCAPM-CAY	Constant	Cay	ΔC	ΔC.cay		
b	0.9051	-67.8	30.6	-14867.3	JT	54.258
t-statistics	1.989	0.852	-0.290	-1.456	*p-value*	*0.0001*
Risk Premium %	0.147	-0.369	0.067	0.016		
t-statistics	0.193	-0.631	0.282	3.441	R-square	46.12%
Shanken-t	0.101	-0.333	0.150	1.819		
GMM-t	0.103	-0.357	0.184	2.124		
CCAPM-C	Constant	ΔC(t-1)	ΔC(t)	ΔC(t).ΔC(t-1)		
b	0.856	-159.3	84.6	-9438.4	JT	59.802
t-statistics	2.827	-1.453	0.952	-1.053	*p-value*	*0.000*
Risk Premium %	2.067	0.856	-0.189	0.006		
t-statistics	3.014	3.112	-0.905	2.717	R-square	40.40%
Shanken-t	1.727	1.806	-0.531	1.579		
GMM-t	1.635	1.858	-0.506	2.037		

Table 10. Results with the 25 Fama-French portfolios sorted by size and book-to-market ratio (FF25) and these 25 portfolio plus the 30 Fama-French industry portfolios (FF55) as the test assets for PROD, PROD-*s*, and PROD-*kθ* (the three version of our productivity-based model) and for FF3, CCAPM-CAY and CCAPM-C, which are respectively the Fama and French (1996) 3-factor model, the Lettau and Ludvigson (2001) conditional (*cay* as conditioning variable) Consumption CAPM, and the Campbell and Cochrane (1999) Consumption CAPM with habit persistence. Italicized numbers represent Shanken-corrected t-statistics. Terms in square brackets indicate 90% confidence intervals around the median based on 5,000 draws from random multivariate normally distributed factors for each group of test assets and with the appropriate number of factors considered for each model. RMSE 30 represents the root mean square error of the 30 industry portfolios determined by the estimated parameters for the FF55 test assets.

PROD	Constant	ε			R-square	JT	RMSE 30
FF25	0.772	0.356			18.18%	95.7	
	0.862	*1.240*			[0.11%, 55.8%]	[35.8, 135.7]	
FF55	0.974	0.267			15.82%	769.0	0.755
	1.437	*1.239*			[0.04%, 18.3%]	[623.0, 947.7]	
PROD-*s*	Constant	ε	ε.*s*				
FF25	0.475	0.869	-0.500		71.53%	28.9	
	0.257	*1.402*	*-1.168*		[3.0%, 69.1%]	[24.7, 112.7]	
FF55	1.083	0.327	-0.060		22.76%	498.2	0.768
	1.473	*1.509*	*-0.528*		[0.8%, 28.0%]	[531.1, 932.5]	
PROD-*kθ*	Constant	ε	ε.θ	ε.*k*			
FF25	1.532	0.867	0.301	-1.700	68.74%	24.5	
	0.939	*1.777*	*0.463*	*-0.925*	[12.5%, 74.3%]	[17.5, 82.4]	
FF55	1.239	0.530	-0.150	-0.540	33.13%	341.6	0.765
	1.399	*2.056*	*-0.406*	*-0.624*	[2.3%, 35.3%]	[462.3, 927.4]	
FF-3	Constant	Mkt	Smb	Hml			
FF25	3.923	-2.416	0.811	1.393	77.57%	81.9	
	3.069	*-1.652*	*1.634*	*2.601*	[12.5%, 74.3%]	[17.5, 82.4]	
FF55	1.947	-0.310	0.619	0.921	31.49%	708.3	0.811
	1.950	*-0.259*	*1.228*	*1.673*	[2.3%, 35.3%]	[462.3, 927.4]	
CCAPM-CAY	Constant	cay	ΔC	ΔC.cay			
FF25	0.1465	-0.3686	0.0667	0.0159	46.12%	54.3	
	0.1005	*-0.3332*	*0.1499*	*1.8186*	[12.5%, 74.3%]	[17.5, 82.4]	
FF55	1.1377	-0.3187	0.1061	0.0028	10.26%	720.4	0.789
	1.5633	*-0.6*	*0.5532*	*0.9639*	[2.3%, 35.3%]	[462.3, 927.4]	
CCAPM-C	Constant	ΔC(t-1)	ΔC(t)	ΔC(t).ΔC(t-1)			
FF25	2.067	0.856	-0.189	0.006	40.40%	59.8	
	1.727	*1.806*	*-0.531*	*1.579*	[12.5%, 74.3%]	[17.5, 82.4]	
FF55	1.417	-0.055	-0.016	-0.002	11.37%	514.8	0.747
	1.719	*-0.295*	*-0.119*	*-1.574*	[2.3%, 35.3%]	[462.3, 927.4]	

Figure 1. Figure 1(a) displays the average excess returns of the 25 portfolios sorted by size and book-to-market ratio. The return data are from Kenneth French's website. Figure 1(b) shows per capita capital and productivity. These data are normalized by subtracting the overall mean and dividing by the standard deviation. The data start at1964Q1 and ends at 2004Q4.

Figure *1(a)*

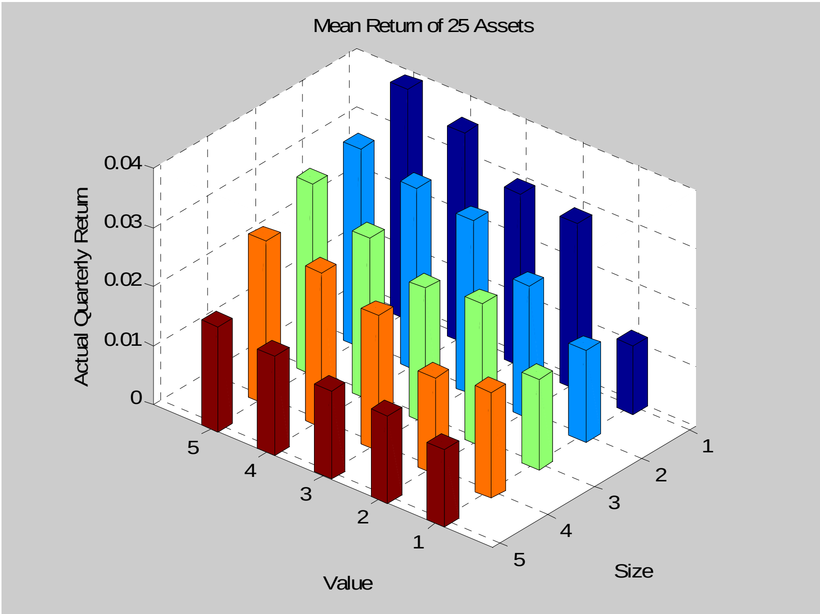

Figure *1(b)*

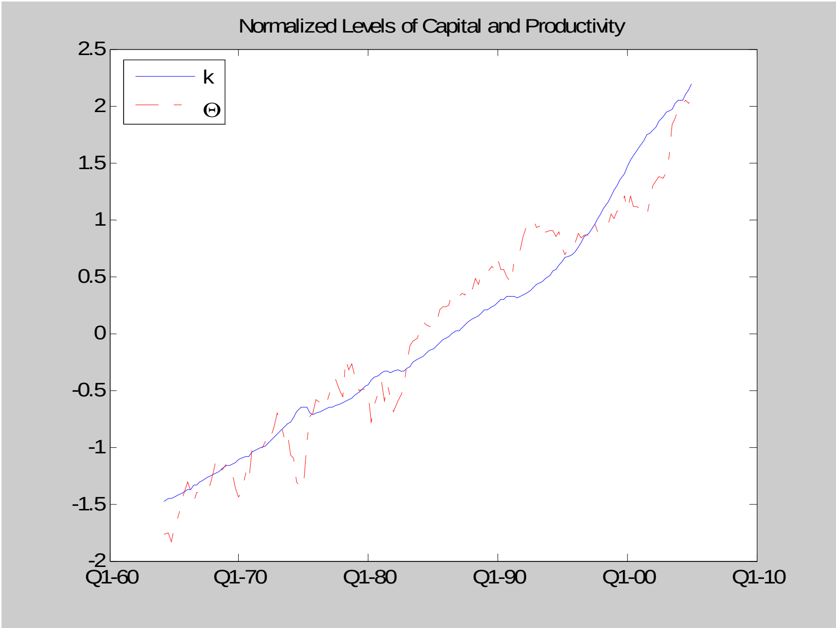

Figure 2. The productivity shock ε (dashed line) and the co-integrating residual *s* (solid line) between capital *k* and productivity *θ* serving as the conditioning/state variable are displayed. NBER business cycle dating is superimposed with the shaded areas indicating recessions. The data are from 1964Q1 to 2004Q4.

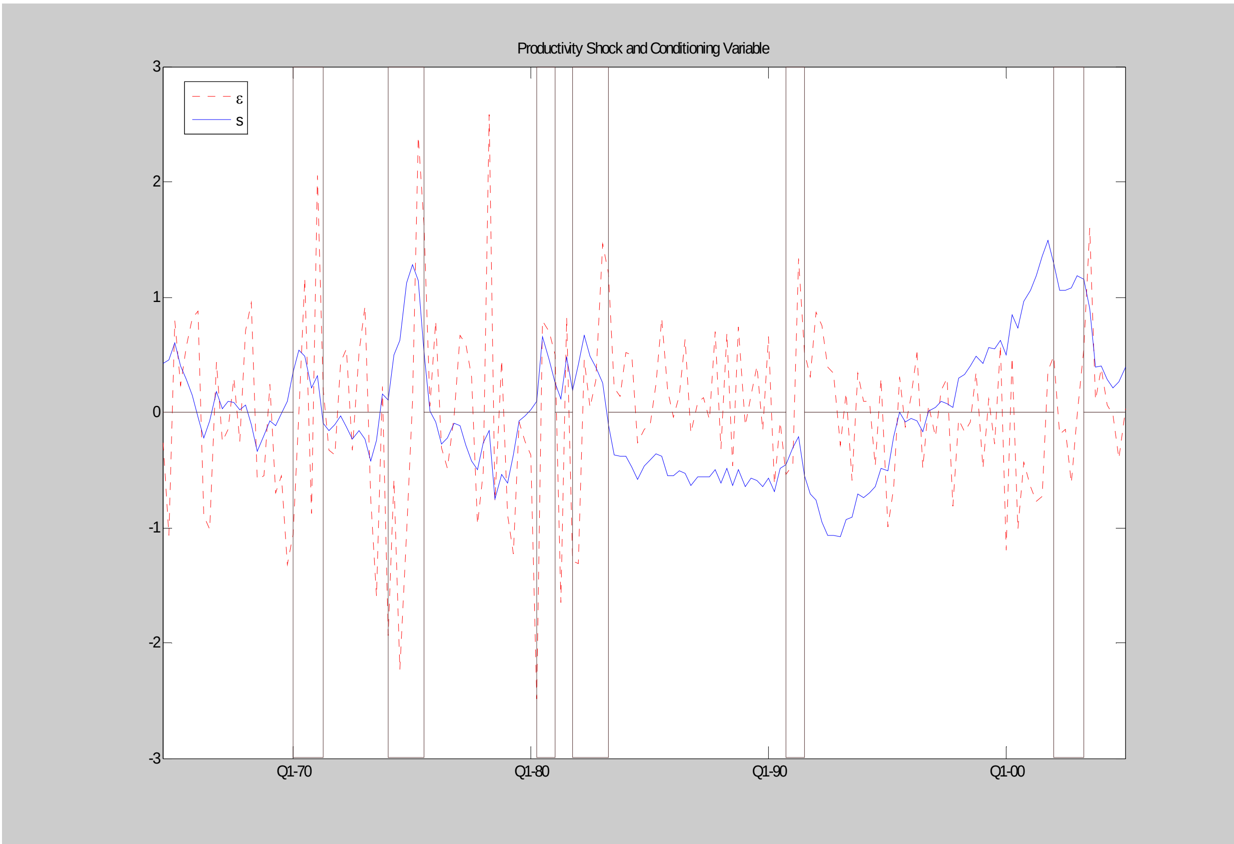

Figure 3. Actual returns and predicted returns for three version of the productivity-based model. Figure *3a* is the unconditional productivity model, Figure *3b* uses the cointegrating residual *s* between *k* and θ as the conditioning variable and Figure *3c* includes both state variables *k* and θ each after subtracting a linear time trend. The data are from 1964Q1 to 2004Q4. The two-digit numbers denote the Fama-French 25 portfolios. The first digit refers to the size quintile (from 1, small, to 5, large) and the second digit refers to the book-to-market quintile (from 1, low book-to-market, to 5, high book-to-market). The figures display the fit for all models by showing the deviations of each test asset's return from that predicted by the model. This deviation is the one-stage GMM pricing error discounted by the expectation of the stochastic discount factor. If there is no pricing error, the actual return equals the predicted return so that all plots locate on the 45-degree line.

Figure *3a*

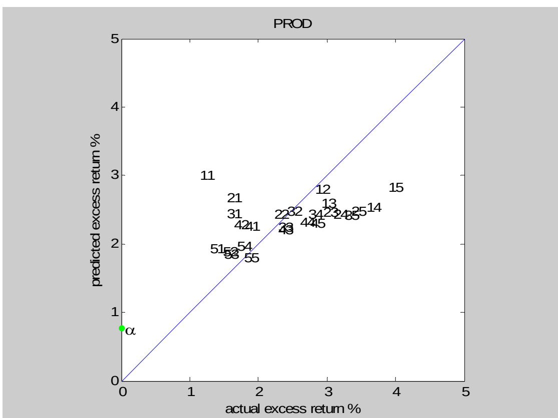

Figure *3b*

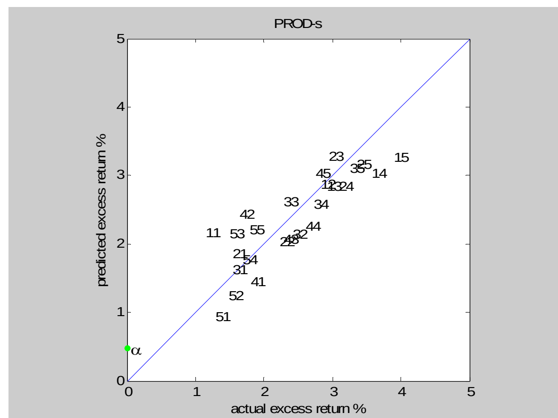

Figure *3c*

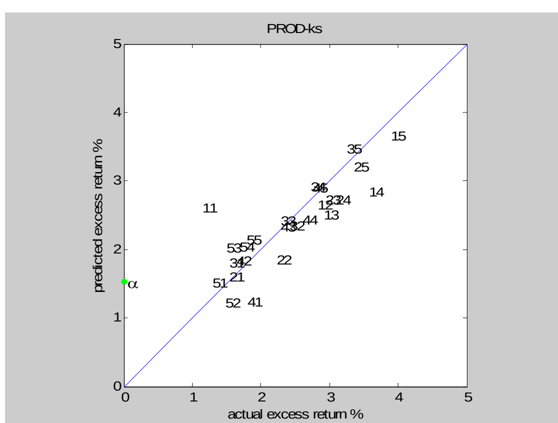

Figure 4. Decomposition of returns into their unconditional—upper left panel—and (the negative of) conditional risk premium—upper right panel—components for the 25 portfolios sorted by size and book-to-market ratio. The lower left panel shows the sum of the unconditional and conditional return components and the lower right panel shows the actual mean returns of the 25 portfolios.

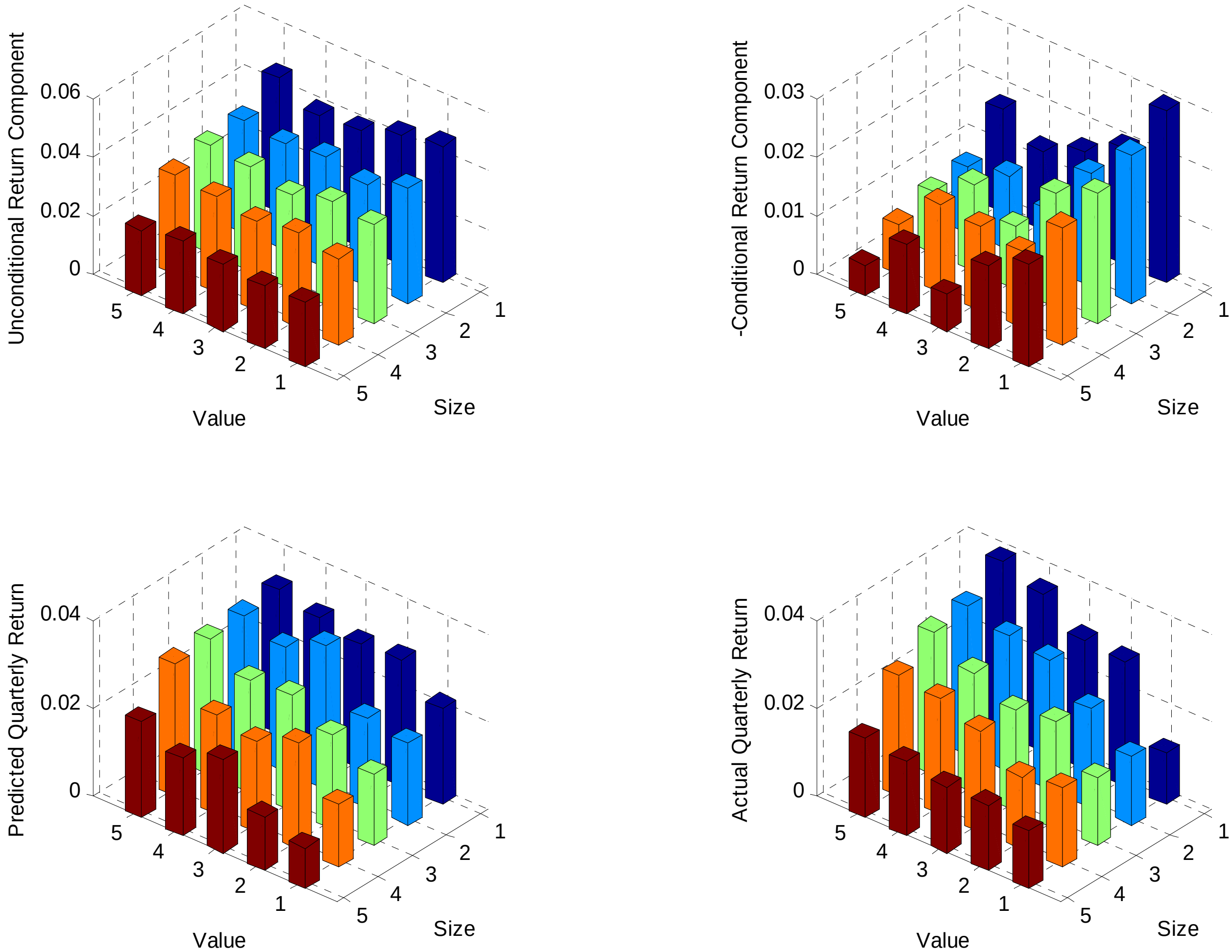

Figure 5. Actual returns and predicted returns for PROD, CCAPM, CAPM and INV which are, respectively, the unconditional productivity model, the consumption-based CAPM, the CAPM and Cochrane's (1996) investment-based asset pricing model. The two-digit numbers denote the Fama-French 25 portfolios. The first digit refers to the size quintile (from 1, small, to 5, large) and the second digit refers to the book-to-market quintile (from 1, low book-to-market, to 5, high book-to-market). The figures display the fit for all models by showing the deviations of each test asset's return from that predicted by the model. This deviation is the one-stage GMM pricing error discounted by the expectation of the stochastic discount factor. If there is no pricing error, the actual return equals the predicted return so that all plots locate on the 45-degree line.

Figure *5a*

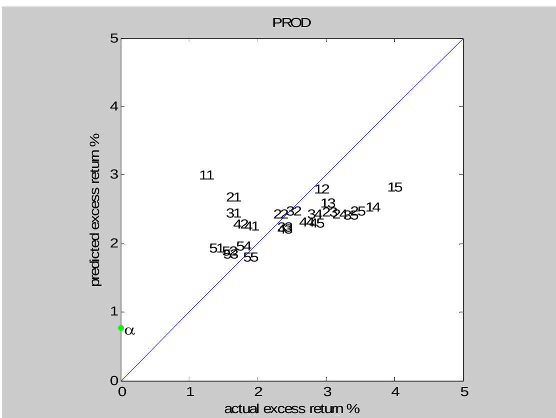

Figure *5b*

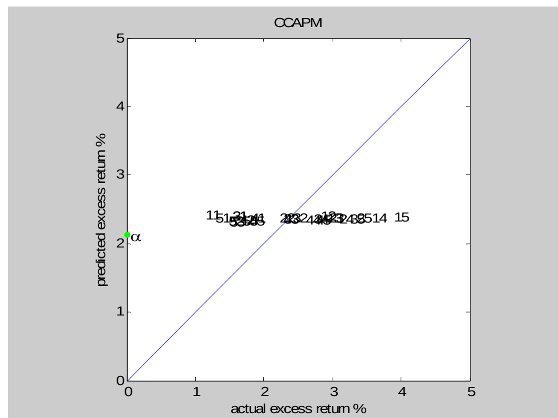

Figure *5c*

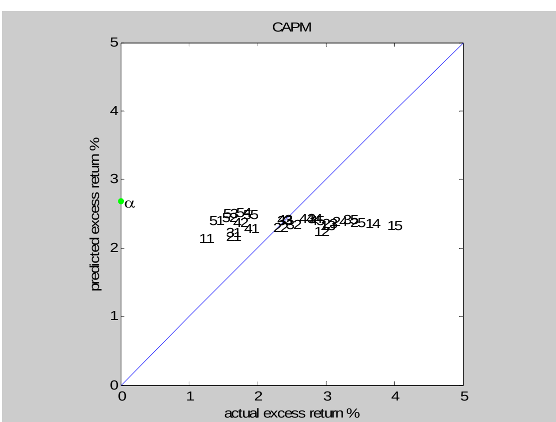

Figure *5d*

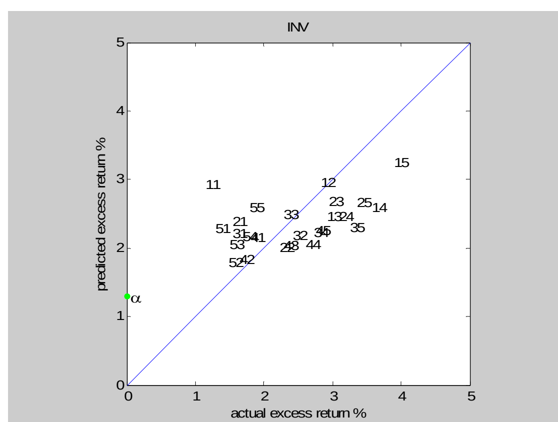

Figure 6. Actual returns and predicted returns for PROD-*s*, FF3, CCAPM-CAY and CCAPM-C which are, respectively, the 2-factor conditional productivity model, the Fama and French (1996) 3-factor model, the Lettau and Ludvigson (2001) conditional (*cay* as conditioning variable) Consumption CAPM, and the Campbell and Cochrane (1999) Consumption CAPM with habit persistence. The two-digit numbers denote the Fama-French 25 portfolios. The first digit refers to the size quintile (from 1, small, to 5, large) and the second digit refers to the book-to-market quintile (from 1, low book-to-market, to 5, high book-to-market). The figures display the fit for all models by showing the deviations of each test asset's return from that predicted by the model. This deviation is the one-stage GMM pricing error discounted by the expectation of the stochastic discount factor. If there is no pricing error, the actual return equals the predicted return so that all plots locate on the 45-degree line.

Figure *6a*

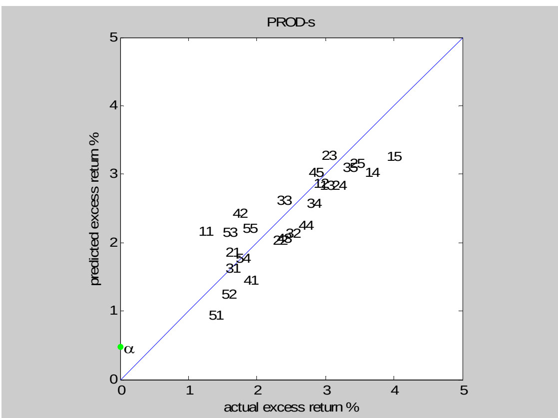

Figure *6b*

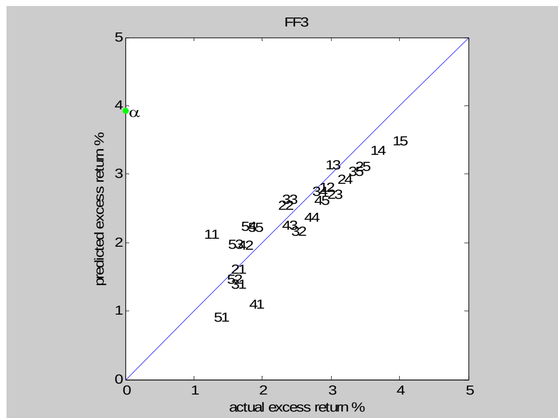

Figure *6c*

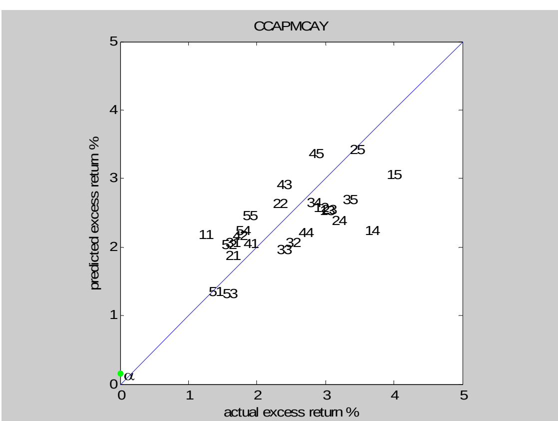

Figure *6d*