TIME PREFERENCE AND LIFE CYCLE CONSUMPTION WITH ENDOGENOUS SURVIVAL

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We provide a structural theory of time preference and derive a functional form of intertemporal preferences by postulating that individuals make their life-cycle consumption choices as if to maximize expected lifetime. This yields a nontime-separable expected utility representation where the inverse of the coefficient of intertemporal substitution exceeds the coefficient of relative risk aversion. The rate of time preference depends on the inverse of expected remaining lifetime and the effect of age on the productivity of consumption in affecting health. The preference formulation is applied in a standard intertemporal consumption model to illustrate the implied life-cycle consumption choices (*JEL* D91, B41).

I. INTRODUCTION

Economic theory has traditionally regarded preferences as given. As a result, there is little guidance for economists on how to formulate intertemporal preferences. The standard approach, derived from Samuelson (1937), is to assume time-separable preferences with a constant rate of time preference. The purpose of this article is to provide a structural theory of intertemporal utility in which the dynamic specification and the rate of time preference are endogenous.

We assume that individuals maximize their longevity—expected life span. Incorporating physical and economic constraints allows us to replace the metaphysical concept of a utility function with the observable concept of a health function. Intertemporal preference is accordingly viewed as the manifestation of whatever design of allocating consumption over time as an input to the health function maximizes expected survival time.

The assumption of maximization of expected life span is consistent with an evolutionary perspective. In line with a growing body of literature, we may view preferences as the end product of natural selection: Subject to physiological constraints, preferences that survive maximize some measure of fitness. Fitness is typically operationalized as number of offspring raised. For instance, Maynard Smith (1982) considers maximization of expected offspring the individual's objective. We adopt the similar but simpler objective of expected life span maximization to focus more directly on time preference issues. We believe that the expected life span objective is a good working approximation for the expected offspring objective.[1] Even beyond the point where individuals are fertile, they

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1. Because evolution is chasing a moving target due to the changing environment, the principle of gene inertia arises. For the human species, this principle implies that impulses designed to maximize fitness in the once-stable hunter-gatherer environment are the impulses that apply today. Increases in lifetime in a hunter-gatherer environment may be identified on a one-to-one basis with increases in reproductive years or, if lifetime lasts beyond reproductive years, increases in the support provided to offspring. See, for instance, Robson and Kaplan (2003) for the view that aged parents aid their offspring via food transfers. Additionally, if a general monotonic relationship between lifetime and surviving offspring is assumed, a change-of-variable establishes (results available on request) that the expected offspring maximization problem can be reduced to one of maximizing expected lifetime in units of reproductive time as long as the health function (to be discussed) is appropriately modified.

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can help increase the fitness of their descendants. In particular, because fitness for humans is as much or more based on mental strength than on physical strength, the elderly can maintain an important role as grandparents. So as people live longer, they can expect to have more and fitter descendants, ceteris paribus.[2]

An alternative evolutionary perspective motivating expected life span maximization is related to cultural learning. Children learn from their parents' generation how to live; the lifestyles that lead to increased life expectancy are more likely to be imitated (either by direct parental guidance or the children's choice). Alternatively, focus on expected life span as a primary goal may be reasonable in modeling subsistence economies, explaining the underlying impediments to development in them. Becker et al. (2001), for instance, emphasize life span considerations to explain that convergence in welfare among countries, though difficult to detect in terms of gross national products, can be observed clearly when longevity improvements are taken into account.

The impact of longevity considerations or survival on intertemporal choice has not received much attention in the literature. Yaari (1965) allows the rate of time preference to vary based on the probability of death; this probability, however, is exogenous.[3] Rogers (1994) applies an idea in Hansson and Stuart (1990)—where the marginal rate of substitution in preferences is set equal to the marginal rate of substitution in "fitness"—to an intertemporal context. His article has a role for bequests and specific implications for how an individual's time preference varies with age. However, it takes a time-additive utility specification as given. Becker and Mulligan (1997) have provided a basic theory of time preference, which is not survival based. They assume that individuals may invest to increase their appreciation of the future, thus endogenously affecting their rates of time preference. As in Rogers, a drawback of the Becker and Mulligan formulation is that it assumes the additively separable intertemporal utility form.

Our theoretical approach derives an intertemporal utility specification that sheds a preliminary theoretical light on debates concerning expected utility, time consistency, separability of consumption decisions, the difference between risk aversion and intertemporal substitution, the effect of health on life-cycle choices, and the factors governing time preference. Section II derives the intertemporal utility specification based on survival-time maximization. Section III considers the properties of the derived utility specification. Section IV provides implications for life-cycle consumption choices, and section V concludes.

II. DERIVATION OF THE INTERTEMPORAL UTILITY FUNCTION

The key assumption here is to impute the maximization of expected lifetime as an individual's sole lifetime goal.[4]

ASSUMPTION 1. *An individual's lifetime objective is to maximize $E(T|\cdot)$, where T is the time of the individual's death.*

Formally, we define a memoryless continuous-time two-state Markov chain $X(t)$: $\mathbf{R}^{+} \rightarrow S$, with $S = \{0, 1\}$. The expectation is taken contingent on the current transitory state $X(0) = 1$; death is defined as the absorbing state $X(t) = 0$. Thus we define the expected lifetime $E(T|\cdot)$ as the mean time until absorption given that $T = inf\{t\colon X(t) = 0 \mid X(0) = 1\}$.

Denote $C(t) \equiv \{c(s)\colon 0 \leq s \leq t\}$ as the consumption history until time t, where $c(s)$: $\mathbf{R}^{+} \rightarrow \mathbf{R}^{+}$ indicates consumption at time s, and $\lim_{t\rightarrow\infty} C(t) \equiv C$ as the infinite horizon consumption path. Then define: $G[t \mid C(t)] \equiv Pr[T \leq t \mid C(t)] = Pr[X(t) = 0 \mid X(0) = 1, C(t)]$ as the probability distribution of being dead by age t given consumption path $C(t)$, and

2. In agricultural economies, for instance, the elderly are vital in land-specific knowledge. They pass on information about the land to the younger generation; this is seen by functional anthropologists as one of the major reasons for why the elderly are taken care of (Ray 1998). Even grandparents that live away from their descendants could play an important role in their descendants' survival by conserving strength so that they are available in case of family emergencies. Note, however, that a limitation of the expected lifetime criterion is that little light can be shed on bequest motives.

3. The uncertain lifetime formulation has been applied by Barro and Friedman (1977), Levhari and Mirman (1977), Davies (1981), Blanchard (1985), and Rosen (1988).

4. Assumption 1 should be contrasted with the assumption made by Karni and Schmeidler (1986) for the purpose of examining risk attitudes: maximization of the end-of-period survival probability. Our approach is different in assuming maximization of expected survival time, which we believe is more appropriate for the purpose of studying intertemporal preferences. Our approach is similar to Karni and Schmeidler's only in modeling competition of the individual against nature while ignoring explicit competition against other individuals.

$g[t \mid C(t)]$ as the associated density.[5] Straightforward derivation yields that the *health-hazard rate* $\lambda[C(t), t] \equiv \lim_{h\downarrow 0}(Pr[X(t) = 0 \mid X(t-h) = 1, \quad C(t)])/h$—the instantaneous probability of dying, *having already lived until time t*—equals

$$(1)\quad \lambda[C(t), t] = g[t|C(t)]/[1 - G[t]C(t)] \\ = -d\ln\{1 - G[t]C(t)]\}/dt.$$

Integrating and taking the antilog on both sides produces

$$(2)\qquad 1 - G[t \mid C(t)] = e^{-\int_0^t \lambda[C(s), s]ds}.$$

Next, relate expected lifetime to the hazard rate. Using integration by parts:

$$(3)\quad E(T \mid C) = \int_0^{\infty} tg[t \mid C(t)]dt \\ = \int_0^{\infty} \{1 - G[t \mid C(t)]\}dt.$$

Combination of equations (2) and (3) produces the intertemporal utility function based on the impersonal evolutionary process that leads to maximization of expected lifetime.

RESULT 1. *Given Assumption 1, an individual's lifetime utility is given as the maximum of*

$$(4)\quad U(C) \equiv E(T \mid C) = \int_0^{\infty} e^{-\int_0^t \lambda[C(s), s]ds} dt.$$

Here T represents time of death and $\lambda[C(t), t]$ *indicates the health-hazard rate at age t, given the consumption stream up to time t.*

III. PROPERTIES OF THE DERIVED UTILITY SPECIFICATION

If the health-hazard rate at t is a function of instantaneous consumption at t only, then this derived utility function belongs to the family axiomatized by Epstein (1983) and thus is a von Neumann–Morgenstern utility function.[6]

To see this more generally in our case, when the health-hazard rate is a function of the stream of consumption until time t as well as of time t itself, we introduce uncertainty other than the hazard of death to shift focus temporarily from considering intertemporal preferences to considering risk preferences. Consider a continuum of possible states belonging to the state space $\Omega \subset \mathbf{R}$. In the above-discussed case where implicitly the state is assumed known, say equal to ω, the agent will determine an infinite horizon consumption plan $C(\omega)$ yielding certain utility $U[C(\omega)] \equiv E[T \mid C(\omega)]$. (Notice that U is deterministic for given ω, even when T is not). When the state is unknown, the (evolutionary) objective is still to maximize expected lifetime, $E(T \mid C)$. But mathematically, $E(T \mid C) = E\{E[T \mid C(\omega)]\} = E\{U[C(\omega)]\}$, where the expectation in the final expression is taken over all $\omega \in \Omega$. We thus have a von Neumann–Morgenstern utility function.

The assumptions needed for the expected utility property (such as the independence axiom) are embedded in the objective we chose to operationalize the concept of survival (Assumption 1). Our contribution is that the assumed linearity in the probabilities in the objective function is not arbitrary in that it is the expected lifetime, resulting from a particular behavioral pattern, that matters. It is outside the scope of the current article—focusing on intertemporal preference—to mathematically justify the assumption of expected lifetime maximization from even more basic principles.[7]

Robson (1996), considering the risk attitudes deriving from an evolutionary process, reaches the same conclusion, supporting expected utility, when risk is idiosyncratic. However, he goes back further in deriving the validity of the expected offspring criterion

5. Ray and Streufert (1993) also assume that survival probabilities depend on the path of consumption but for a time-additive utility function.

6. Bergman (1985), Epstein and Hynes (1983), Obstfeld (1990), and Uzawa (1963) also discuss members of the class axiomatized by Epstein (1983). These authors, however, do not examine the particular form that we derive here.

7. An evolutionary motivation, however, is that the independence axiom (considered the most controversial of the axioms guaranteeing the expected utility property) should hold because taking account of the realizations of unrealized alternatives is unproductive and has no survival value. The survival value attached to a particular consumption path should not depend on what would happen if another consumption path were realized, as we implicitly assume via Assumption 1.

used by Maynard Smith (1982) and others (including us).[8]

Returning to issues of dynamic preference specification, and accordingly dropping for simplicity all uncertainty other than the hazard of death, we now show that our utility function in equation (4) also implies time consistency. Factoring the right-hand side of equation (4) yields:

$$\int_0^\infty e^{-\int_0^\tau \lambda[C(s),s]ds} d\tau = \int_0^t e^{-\int_0^\tau \lambda[C(s),s]ds} d\tau + e^{-\int_0^t \lambda[C(s),s]ds} \left(\int_t^\infty e^{-\int_t^\tau \lambda[C(s),]ds} d\tau \right)$$

Or, using equation (4) both for the first and for the last term,

$$E(T \mid C) = a[C(t), t] + b[C(t), t] E(T - t \mid C, t),$$

where the conditioning information t is shorthand for $X(t) = 1$ so that $E(T \mid C) \equiv E(T - 0 \mid C, 0)$. Thus the specific form here allows us at each point in time t to maximize $E(T - t \mid C, t)$. Clearly, the decisions based on the continuation at time t of the policy based on preferences at time 0 are equivalent to the decisions made at time t based on preferences at time t—anticipated preference reversals do not occur and preferences are time consistent:

$$\underset{\{C(s)\}|_t^\infty}{argmax}\ E(T \mid C) = \underset{\{C(s)\}|_t^\infty}{argmax}\ E(T - t \mid C, t)$$

Deaton (1992, 15) states that time-inconsistent preferences are irrational. This is not obvious given a typical view of rationality as "behavior consistent with the objectives"—complex objectives may, in principle, allow for any type of behavior. To the extent that survival mechanisms can only support objectives like maximization of expected lifetime, our approach provides a rationale for marking time inconsistent behavior as irrational.

A further property—that preferences are not time separable—follows directly from equation (4). It is straightforward to show that the marginal utility of consumption at time t depends on future levels of consumption (as is apparent from equation [5], for instance). The absence of time separability is consistent with the opinions of many (for instance, Lucas 1978, 1444) that there is no rationale other than convenience for assuming a time-additive utility specification. Uzawa (1963) previously considered without derivation a nontime-separable form related to ours but with the rate of time preference given ad hoc, as a function of consumption in different periods. In Ryder and Heal (1973), the utility function is assumed to depend on a weighted average of past consumption levels, with weights declining exponentially into the past.[9]

The above consequences are summarized as follows.

RESULT 2. *The preference specification* $U(C)$ *in equation* (4) *consistent with maximizing expected lifetime according to Assumption* 1: (*a*) *has the expected utility property*; (*b*) *is time consistent*; *and* (*c*) *is not time separable.*

Further properties of the derived utility functional in equation (4) will be obtained under a simpler specification of the health-hazard rate.[10]

ASSUMPTION 2. *For all* $t \in [0, \infty)$ *the hazard rate depends only on age t and current consumption* c(t): $\lambda[C(t), t] = \lambda[c(t), t]$. *It is twice continuously differentiable and a positively valued, negatively sloped, and strictly convex function of* $c(t)$.

8. Robson additionally finds that selection in the context of aggregate shocks may invalidate the expected offspring criterion: The effect of a shock on an individual is different when all members of a particular risk-attitude type are affected by the shock than when instead only the individual is affected (the aggregate risk lowers the expected growth rate more than the idiosyncratic risk). Robson interprets this result as generating nonexpected utility behavior (the axiom of reduction of compound lotteries breaks down, not the independence axiom), but one could alternatively consider expected-utility maximizing individuals as endowed with altruistic feelings toward their group, causing them to be more risk averse in the face of aggregate risk.

9. More recent applications of the nontime-separable utility specification include the work of Bergman (1985), Obstfeld (1990), and Shi and Epstein (1993).

10. In a more general specification, the health hazard could also depend on the lagged health hazard to add realism. However, our specific and methodological points are most easily made using the simpler specification of Assumption 2.

Given equation (4) and the assumed hazard rate specification, a change in consumption at time t implies the following Volterra derivative denoted by$'$ (for a similar use of the Volterra derivative see, for instance, Ryder and Heal 1973; Epstein and Hynes 1983):

$$(5) \quad U'(t) = -\lambda_c[c(t), t](1 - G[t \mid C(t)]) \times E(T - t|C,t),$$

where

$$(1-G[t|C(t)])E(T-t|t) = \int_t^{\infty} e^{-\int_0^{\tau} \lambda[C(s),s]ds} d\tau.$$

As in the following, subscripts represent partial derivatives. Thus $-\lambda_c$ indicates the marginal impact of additional consumption in reducing the health hazard, which is multiplied by the probability of being alive at time t and expected remaining lifetime, the latter representing the loss at sudden death.

Some basic properties of the utility functional follow readily from equation (5). Consider a discrete-time version of equation (4). Because utility is monotonically increasing in each of its arguments from equation (5), it follows directly that the discrete version of the utility specification in equation (4) is monotonic and thus quasi-concave. The continuity of $\lambda()$ from Assumption 2 ensures that the continuous-time utility functional is quasi-concave as well. As a result, the indifference curves for consumption at two separate instants in time are convex:

RESULT 3. *Given Assumption* 2, *the utility functional in equation* (4) *is monotonic and quasi-concave and has convex indifference curves.*

Next define the discount rate following Epstein and Hynes (1983) as,

$$(6) \quad \rho(t) = \frac{U'(t-h)}{U'(t)} - 1 = -\partial \ln U'(t)/\partial t$$

with $c(t) = c(t-h)$ and for $h \downarrow 0$ Equation (6) captures the basic notion of time preference in continuous time: all else (i.e., consumption) equal, by which fraction is the marginal utility from consumption at some point in time, $t - h$, higher than the marginal utility from consumption at a slightly later point in time, t. This time preference concept is a marginal concept—applying to time preference at a point in time. As time preference may change over time, the average concept—considering the rate of time preference over a longer period—will in general be different.

Differentiating the log of the right-hand side of equation (5) with respect to time t, equation (6) yields after some cancellations:

$$(7) \quad \rho(t) = \left[\int_t^{\infty} e^{-\int_t^{T} \lambda[C(s),s]ds} dT\right]^{-1} - \lambda_{ct}[c(t), t]/\lambda_c[c(t)].$$

Equation (7) implies

RESULT 4. *Given Assumptions* 1 *and* 2, *an individual's rate of time preference* (*discount rate*) *is equal to*

$$(8) \quad \rho(t) = [1/E(T - t \mid C,\, t)] - \lambda_{ct}[c(t), t] / \lambda_c[c(t),\, t].$$

With t a particular time period during which the individual is alive

The derivation follows from equation (7) and the definition in equation (4).

Assuming that the health-hazard rate is separable in the consumption and age components so that $\lambda_{ct}[c(t), t] = 0$ in equation (8), individuals with a higher life expectancy have a longer horizon and thus a lower rate of time preference as they rationally put more weight on future events. When $\lambda_{ct} \neq 0$, age affects the productivity of consumption in affecting health so that an extra factor determines time preference: If consumption becomes, say, more crucial for health with age, $\lambda_{ct} < 0$, then, all else equal, current consumption should fall compared to future consumption, meaning more patience—smaller $\rho(t)$. In general, the sign of λ_{ct} is an empirical question. Consumption could become more important for health with age as better care is needed to survive (such as medical needs, good shelter and climate, appropriate foods, etc.) or, at some point, could become less important for health as old age takes its toll whatever the consumption inputs.

Taking the result a little more seriously than is intended, one may obtain the numerical value for the average rate of time preference from equation (8), when λ_{ct} is set to zero. Based on the instincts surviving from hunter-gatherer times, the conditional life expectancy of the average individual living through early childhood may lie around 30–35 years left to live.[11] The rate of time preference from equation (8) should then be around 3%, which appears to be in the ballpark compared to the actual numbers.[12]Again assuming constant health effects of consumption with age (or controlling for age), some confirmation of Result 4 is provided by Leigh (1986), who finds that—controlling for income—African Americans, who as a group have a lower life expectancy, also have a significantly higher rate of discount (a result confirmed by Cropper et al. 1994).

Some further results easily follow from Result 4:

RESULT 5. *Given Assumptions* 1 *and* 2, *if the effect of consumption on health is independent of age*, *wealthier individuals cannot have a higher rate of time preference.*

Proof. An increase in wealth cannot decrease an individual's maximum utility level. Thus, from equation (1), expected lifetime rises (or remains unchanged) which lowers (or maintains) the rate of time preference from equation (8) when λ_{ct} equals 0.

Given Result 5, it is easy to imagine a cycle of poverty. As an individual becomes poorer, he or she also rationally becomes more myopic, leading to relatively higher consumption, exacerbating the degree of poverty. Result 5 is confirmed empirically by Lawrance (1991) and Viscusi and Moore (1989). Lawrance, using panel data, finds a rate of time preference of poorer households that is 3–5% higher than that of wealthier households. Viscusi and Moore find that households with lower earning potential (lower lifetime wealth) have a higher rate of time preference than those with higher earning potential.

An additional result provides the circumstance under which the rate of time preference equals the hazard rate as assumed for instance in Blanchard (1985).

RESULT 6. *If consumption is constant and health does not depend on age then the rate of time preference is constant and equal to the health-hazard rate.*

Proof. In equation (7) keep consumption constant and pull through the integral to obtain:

$$(9) \qquad \rho(t) = \left[\int_t^\infty e^{-\lambda(\tau-t)}d\tau\right]^{-1} = \left[\int_0^\infty e^{-\lambda\tau}d\tau\right]^{-1} = \lambda.$$

The last equality holds because the term in square brackets represents the expected value of the exponential distribution with parameter λ. This result can be compared to that of Yaari (1965) who finds that with uncertain lifetime, the effective rate of time preference is equal to an assumed subjective rate of time preference plus the (exogenous) mortality rate. In Result 6, the (effective as well as subjective) rate of time preference is equal to the mortality rate.

A limitation of the standard time-separable specification of utility is that the coefficient of risk aversion must equal the inverse of the elasticity of substitution. In the appendix we derive that

$$(10) \qquad R[c(t),t] = [c\{\lambda_{cc}[c(t),t] - \lambda_c^2[c(t),t]\}] / -\lambda_c[c(t),t]$$

11. Kaplan et al. (2000) and Robson and Kaplan (2003) display the annual mortality rates of present-day hunter-gatherers such as the Aché of South America. The mortality rates are U-shaped with age, lying below 2% from later childhood until around age 45 and increasing steeply only beyond age 65 or so. This implies an expeced lifetime of those living beyond early adulthood of around 55 years. However, the frequency-weighted *average remaining* lifetime of all adults would be much lower, somewhere around 30–35 years.

12. Rogers (1994) obtains a slightly lower number based on population growth, average generation length, and the fraction, 0.5, of shared genes between parent and offspring.

where $R[c(t), t]$ represents the coefficient of relative risk aversion at a consumption level for time t. Defining $\sigma[c(t), t]$ as the coefficient of intertemporal substitution, the appendix obtains

$$(11) \quad 1/\sigma[c(t),t] = c\lambda_{cc}[c(t),t]/-\lambda_c[c(t),t]$$

Straightforward comparison of equations (10) and (11) produces the following result:

RESULT 7. *The inverse of the coefficient of intertemporal substitution exceeds the coefficient of relative risk aversion.*

The intuition is that the coefficient of intertemporal substitution captures the incentive to smooth the hazard of death over time, which depends on the curvature of the hazard rate. The coefficient of relative risk aversion, on the other hand, relates to the curvature of overall utility, which is less than the curvature of the hazard rate: A decrease in consumption at t lowers expected lifetime, which dampens the overall effect on marginal utility due directly to a higher hazard of death at t (because the opportunity cost of death is equal to expected remaining lifetime).

An example may help illustrate some of the advantages of the approach. Consider an isoelastic hazard rate, $\lambda(c,t) = [\alpha(t)/\gamma(t)]\, c^{1-\gamma(t)}$, where $\gamma(t) > 1$ is required for convexity. Then $1/\sigma(c,t) = \gamma(t)$ and $R(c,t) = \gamma(t) - \alpha(t)c^{1-\gamma(t)}$. Changes in the parameter path $\alpha(t)$ affect risk aversion without affecting intertemporal substitution. In this example, risk aversion decreases as consumption falls because $\gamma(t) > 1$. For very low levels of consumption, it even pays to seek risk and gamble. Seeking risk may be optimal in desperate situations, but it is easy to show that positive risk aversion can be guaranteed for all consumption levels if the health-hazard rate is equal to any monotonically increasing, concave transformation of the function $\gamma - \ln(c - \alpha)$ for $\alpha, \gamma > 0$.

IV. IMPLICATIONS FOR LIFE CYCLE CONSUMPTION

To illustrate the implications of our preference formulation for consumption plans over the life cycle, we incorporate the derived utility function in a standard intertemporal consumption model.

Optimal Consumption

Continuity of the health-hazard function and the natural bounds on consumption guarantee that the savings problem for an expected-lifetime optimizer may be formulated as follows:

$$(12) \quad V[W(t),t] = \max_{\{c(s)\}_{s=t}^{\infty}} E(T-t|C,t),$$

$$\text{s.t. } \dot{W}(t) = rW(t) - c(t),\, W(0) = W_0,$$

$$W(t) \geq 0 \;\forall t$$

$$E(T-t|C,t) = \int_t^{\infty} e^{-\int_t^{\tau} \lambda[c(s),s]ds} d\tau$$

At time t the consumer chooses the consumption path to maximize the conditional expected survival time, $E(T-t \mid C,t)$, as given in equation (5). Maximization is subject to the constraint that lifetime wealth remains non-negative, where the change in wealth equals the return on wealth minus consumption. For simplicity we assume that the consumer's wealth accumulates via interest only and that no income is received (alternatively, one could interpret wealth as lifetime wealth, including the present value of future income).

The appendix yields the Hamilton-Jacobi equation of dynamic programming for the consumption problem:

$$(13) \quad 0 = \max_c [1 - \lambda(c,t)V(W,t) + V_W(W,t)(rW - c) + V_t(W,t)]$$

Subscripts indicate partial derivatives, as before. The current payoff equals 1 (period of life) minus the hazard of current death times the loss of the remaining life expectancy. The future is affected by the changes in the two state variables: the change in wealth times the effect of wealth on expected remaining lifetime plus the change in time (equal to 1) times the effect of time on expected remaining lifetime. The last term underscores that health changes (deteriorating typically) over time for given wealth.

The optimal choice of current consumption is given by the following first-order condition for the dynamic programming problem of equation (13) subject to the equation of motion and nonnegativity condition for wealth in equation (12),

$$(14) \qquad -\lambda_c(c,t)V(W,t) - V_W(W,t) = 0.$$

The increase in the probability of survival, $-\lambda_c$, times the expected remaining life is traded off against the loss of life expectancy due to the decrease in wealth necessary to finance the additional consumption.[13].

It is easily established from Result 4 together with equations (12) that the rate of time preference is related to the inverse of the value function: $\rho(t) = (1/V) - \lambda_{ct}/\lambda_c$. An alternative expression for the rate of time preference follows from equations (13) and (14):

$$(15) \qquad \rho(t) = \lambda + \lambda_c(rW - c) - (V_t/V) - \lambda_{ct}/\lambda_c.$$

The rate of time preference accounts for four survival factors—in order, the hazard rate, the decrease in survivability when wealth falls, the decrease in survivability due to deteriorating health, and the increase with age of consumption's productivity in improving health. Equation (15) is useful because it clarifies why the time preference rate is not equal to the hazard rate λ here, as is the case for the simple time-additive case with uncertain lifetime of Yaari (1965) (when subjective time preference is ignored) or for our specification under the conditions of Result 6.

The envelope condition for wealth based on equation (13) becomes:

$$(16) \qquad V_{WW}\dot{W} + V_{Wt} = (\lambda - r)V_W.$$

Next, totally differentiate the first-order condition in equation (14) with respect to time:

$$(17) \qquad -\lambda_{cc}V\dot{c} - \lambda_{ct}V - \lambda_c V_W \dot{W} - \lambda_c V_t = V_{WW}\dot{W} + V_{Wt}.$$

Employing equation (16) for the right-hand side of equation (17), and equations (11), (14), and (15) gives a standard expression for the change of consumption with time:

$$(18) \qquad \dot{c}/c = \sigma[r - \rho(t)].$$

Consumption has more of a tendency to grow over time as the wealth benefit of postponement, r, is higher; the private rate of time preference, $\rho(t)$, is lower. The endogenous expression for $\rho(t)$ in equation (15) distinguishes our results from others.

Life Cycle Consumption Implications

Consumption over Time. In standard consumption models with a constant interest rate and constant exogenous rate of time preference, equation (18) allows only monotonically increasing or decreasing consumption over the life cycle. In our model, however, a variety of interesting life cycle patterns may arise. Assume, for instance, that the health-hazard rate is separable in consumption and age so that $\lambda_{ct}[c(t), t] = 0$. Also assume that in general equilibrium, market forces set the interest rate equal to the average discount rate in the population, $r = \rho_{avg}$. Because for reasonable health-hazard specifications it must be that life expectancy eventually falls with age, we know that $\rho(t)$ eventually increases with age, as follows from Result 4. Thus, for a typical individual, the discount rate moves from below average to above average so that equation (18) implies that consumption growth is positive early in the life cycle but eventually becomes negative. The result is a hump-shaped consumption pattern over the life cycle, not unlike typical consumption patterns in present-day society.[14]

Consumption with Perfect Annuity Markets. Yaari (1965) obtains an equation similar to equation (18). One difference, as pointed out earlier, is that Yaari's rate of time preference equals a subjective rate of

13. The transversality condition for this infinite horizon problem is $[1 - G(T)]W(T)V_W(W, T) \to 0$ as $T \to \infty$.

14. A specific example with a health-hazard function that has a U-shaped hazard rate is available in Acharya and Balvers (2003). The example illustrates, for instance, an endogenous cycle of poverty and implies that life expectancy peaks before health peaks and that healthier people are more risk averse (because they have more to lose).

time preference plus an exogenous mortality rate, $\rho(t) = \bar{\rho} + \lambda$, instead of the rate of time preference as given in equation (15). A further difference depends on whether a perfect annuity market is assumed to exist. With such a market the decision problem in equation (12) changes. Yaari argues that

$$(12') \qquad \dot{W}(t) = (r + \lambda)W(t) - c(t)$$

because one can freely buy and sell actuarially fair annuities that pay a fixed amount until the individual's death. The competitive return on such an annuity equals $r + \lambda$: interest plus a payment for the hazard of the annuity ending when the individual dies. In the absence of a bequest motive, it is optimal for the individual to hold such annuities only.

Using (12′) instead of (12) in the optimization problem, we can derive easily that

$$(18') \qquad \dot{c}/c = \sigma[r + \lambda - \rho(t)].$$

In Yaari's model we get $\dot{c}/c = \sigma[r + \lambda - \bar{\rho} - \lambda]$ so that even exogenous changes with age in mortality rate λ have no impact on consumption behavior over the life cycle. In our model we obtain from equation (15) and after canceling the λ's that $\dot{c}/c = \sigma[r - \lambda_c \dot{W} + (V_t/V) - \lambda_{ct}/\lambda_c]$, with $\dot{W}$ given in equation (12′). Clearly, a variety of interesting consumption patterns over the life cycle is possible, even with perfect annuity markets.

The interesting issue arises whether perfect annuity markets could exist in our context: Individuals in our model can manipulate mortality rates so that a moral hazard issue arises. Consider, for instance, the annuity contract return $r + \lambda$ set for the (limited) duration of the contract based on observable characteristics (wealth and age in our model). Then if the annuity is sold (individual receives a fixed amount in exchange for the obligation to pay $r + \lambda$ each period until death or the final period of the contract) the individual may choose to consume less than planned in absence of the contract, raising mortality for the duration of the contract, but accumulating wealth. If the annuity is purchased (individual pays a fixed amount in exchange for the benefit of receiving $r + \lambda$ each period until death or the final period of the contract) the individual may consume more than optimal in absence of the contract, lowering mortality for the duration of the contract and raising future mortality rates, which should yield higher annuity returns on future contracts.

Public Pensions. The way to avoid such moral hazard is to issue infinite-maturity annuity contracts only, which indeed is common in practice. Public pension plans are one way of providing infinite maturity annuity contracts. However, because the pension benefits are anticipated and are not based on individual observable wealth characteristics, a moral hazard issue may still arise. To examine this issue we assume that private annuity markets do not exist (otherwise individuals could just privately undo the restrictions imposed on them by the public pension plan).

Without annuity markets, the decision problem of equation (12) applies again. There are two direct effects of introducing a fully funded public pension plan. First, this plan has an effective return on public funds of $r + \lambda$. So the amount of tax needed is less, and individuals receive in effect a return of $r + \lambda$ on their tax payments. Second, if we assume again that $\lambda_{ct}[c(t), t] = 0$ then we know that $\rho(t)$ eventually increases with age. Thus, equation (18) implies that spending plans have wealth drawn down to zero in finite time. The public pension plan accordingly is effective if the individual is still alive at that time. Put another way, if we ignore the first effect, the individual faces lower utility (expected lifetime) because he or she is impelled to consume more later in life, whereas optimal plans would have an individual consume wealth earlier. These effects are present in the Yaari framework without annuity markets as well, as long as mortality rates rise (exogenously) with age. In our model, the moral hazard issue adds complications that are not easy to evaluate and are outside the scope of this article. It is possible that the guaranteed pension alters consumption plans so that wealth is drawn down earlier to maximize the chance that the individual survives to cash in on the public plan. This would constitute a new avenue, when survival is endogenous, by which a public pension plan discourages saving.

V. CONCLUSION

We have provided a theoretical basis for dynamic utility specifications. The survival-oriented rationality of the individual objective function implies time consistency; the derived utility function, although recursive, is not time-separable. It must be of the von Neumann–Morgenstern variety, however. The theory provides insights into life cycle consumption choices by showing that the rate of time preference varies in an intuitive manner with changes in conditional lifetime, initial wealth, age, and the marginal productivity of consumption in affecting health. Consumption over the life cycle is likely to display a hump-shaped pattern, which contrasts with traditional consumption models that typically yield a monotonic consumption path.

Kacelnik (1998) states: "Neither animals nor humans are likely to be driven directly by the maximization of fitness, but we may understand the psychological mechanisms that do control their behaviour by asking about the fitness consequences of different courses of actions." This statement characterizes our basic approach. Operationalizing 'maximization of fitness' as we do in terms of maximization of expected lifetime, however, has some important shortcomings as an evolutionary motivation: It ignores the trade-off between survival and fertility, strategic interactions between individuals, and altruism among kin leading, for instance, to a bequest motive. Extending our approach to address these simplifications may yield further interesting results.

Explanation of standard anomalies may of course require other extensions of the approach, for instance by altering assumption 2 to allow the consumption history to affect current health. Take the observation that individuals prefer to delay pleasant events and like to accelerate unpleasant events as discussed by Loewenstein (1987). The survival-based explanation would be that an individual currently in good health would prefer to deal with unpleasant (i.e., potentially hazardous to life) events quickly, when bad outcomes can easily be absorbed; whereas pleasant events should be postponed so that they may benefit the individual at a potentially vulnerable time.

APPENDIX

Derivation of Equation (10)

We can take the Volterra derivative of equation (5) to obtain

$$\text{(A-1)} \quad U''(t) = -\lambda_{cc}(c,t)\int_t^{\infty} e^{-\int_0^{\tau}\lambda(c,s)ds}\,d\tau + [\lambda_c(c,t)]^2\int_t^{\infty} e^{-\int_0^{\tau}\lambda(c,s)ds}\,d\tau.$$

Again recalling (5), the standard Arrow-Pratt measure of relative risk aversion, using (A–1), is

$$R(c,t) \equiv -U''(t)c/U'(t) = \left(c[\lambda_c^2(c,t) - \lambda_{cc}(c,t)].\times\int_t^{\infty} e^{-\int_0^{\tau}\lambda(c,s)ds}d\tau\right)\Big/\left(-\lambda_c(c,t)\int_t^{\infty} e^{-\int_0^{\tau}\lambda(c,s)ds}d\tau\right).$$

Canceling the integral expressions produces equation (10) in the text.

Derivation of Equation (11)

The elasticity of the marginal rate of substitution between consumption at time t_2 and t_1 (with $t_2 > t_1$) can be expressed (see Silberberg 1990, 288) as:

$$\sigma \equiv (-U'(t_1)U'(t_2)[U'(t_1)c(t_1) + U'(t_2)c(t_2)] / (c(t_1)c(t_2)[U'(t_2)^2U''(t_1) - 2U'(t_1)U'(t_2) \times U''(t_1,t_2) + U'(t_1)^2U''(t_2)].$$

Taking Volterra derivatives based on equation (5) for $t_2 > t_1$:

$$\text{(A-2)} \quad U''(t_1,t_2) = \lambda_c(c,\ t_1)\,\lambda_c(c,\ t_2)\int_{t_2}^{\infty} e^{\int_0^{\tau}\lambda(c,s)ds}\,d\tau.$$

Let $t_2 \downarrow t_1$ so that, for continuous $c(t)$, $c(t_1) \to c(t_2)$ to ensure that $U'(t_1) \to U'(t_2)$. Then (A-2) becomes

$$U''(t,t) = \lambda_c^2(c,t)\int_t^{\infty} e^{-\int_0^{\tau}\lambda(c,s)ds}\,d\tau.$$

The elasticity of substitution then equals:

$$\sigma[c(t),t] = [2U'(t)^3c(t)]\Big/\left(2[U'(t)c(t)]^2 \times \left[U''(t) - \lambda_c^2(c,t)\int_t^{\infty} e^{-\int_0^{\tau}\lambda(c,s)ds}d\tau\right]\right).$$

Using equation (5) and (A–1) yields the inverse of equation (11):

$$\sigma[c(t),t] = -\lambda_c[c(t),t]/c(t)\,\lambda_{cc}[c(t),t]$$

Derivation of Equation (13)

Maximization at time t of $E(T\,|C)$, which is given by equations (3) and (4), subject to the wealth constraint $W(0) = W_0$, $\dot{W}(t) = rW(t) - c(t)$, $W(t) \geq 0$ for all t, leads to the following dynamic programming problem:

$$\text{(A-3)} \quad J[G(t), W(t), t] = \max_{\{c(s)\}_{s=t}^{\infty}} \left[\int_t^{t+\Delta t} [1 - G(s)]ds + \int_{t+\Delta t}^{\infty} [1 - G(s)]ds \right.$$

$$\text{(A-4)} \quad \text{s.t. } \dot{W}(t) = rW(t) - c(t), \; W(t) \geq 0$$

$$\text{(A-5)} \quad G(t) = 1 - e^{-\int_0^t \lambda[c(s),s]ds}$$

Rewrite (A-3):

$$\text{(A-6)} \quad J[G(t), W(t), t] = \max_{\{c(s)\}_{s=t}^{t+\Delta t}} \{[1 - G(t)]\Delta t + J[G(t + \Delta t), W(t + \Delta t), t + \Delta t]\}.$$

The linear approximation, that becomes exact as $\Delta t \to 0$, produces:

$$\text{(A-7)} \quad 0 = \max_c [1 - G + J_G(G, W, t)\dot{G} + J_W(G, W, t)\dot{W} + J_t(G, W, t)].$$

It follows from equation (5) evaluated at time t and decision problem (12) plus the definition in (A-5) that $J(G,W,t) = (1 - G)V(W,t)$ and from (A-5) that $\dot{G} = \lambda(c, t)(1 - G)$. Because $J_W = (1 - \text{G})\ V_W$, $J_G = -V$, $J_t = (1 - G)V_t$ and $J_t = (1 - G)V_t$ we can rewrite (A-7) to obtain:

$$\text{(A-8)} \quad 0 = \max_c [(1 - G) - \lambda(1 - G)V + (1 - G)\ V_W\ \dot{W} + (1 - G)\ V_t].$$

Dividing through by 1 − G produces equation (13) in the text.

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